The Mathematics of the Chinese Calendar

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1 Introduction

Chinese New Year is the main holiday of the year for more than one quarter of the world's population. In addition to China, Taiwan, Hong Kong, Macau and Singapore it is also a public holiday in South Korea, Indonesia, Malaysia, Brunei, Laos, Vietnam and Mauritius. Very few people, however, know how to compute its date. For many years I kept asking people about the rules for the Chinese calendar, but I was not able to find anybody who could help me. Many of the people who were knowledgeable about science felt that the traditional Chinese calendar was backwards and superstitious, while people who cared about Chinese culture usually lacked the scientific knowledge to understand how the calendar worked. In the end I gave up and decided that I had to figure it out for myself. This paper is the result.

The rules for the Chinese calendar have changed many times over the years. This has caused a lot of confusion for people writing about it. Many sources describe the rules that were used before the last calendar reform in 1645, and some modern sources even describe the rules that were used before 104 BCE! In addition, the otherwise authoritative work of Needham ([38]) almost completely ignores the topic. (See however the article by Cullen ([7]).) For many years, the only reliable source in English was the article by Doggett in the Explanatory Supplement to the Astronomical Almanac ([12]), based on unpublished work of

Liu and Stephenson ([28]). But thanks to the efforts of Dershowitz and Reingold ([11]), correct information and computer programs are now easily available. Among Chinese sources, my favorite is the book by Tang ([48]).

Even though these sources give the basic rules, the Chinese calendar is such a rich subject that there's still a lot left to study. The basic principles are fairly simple, but in some exceptional cases it can get extremely complicated. The year 2033 is such an exceptional case and until the early 1990's, all the Chinese calendars placed the leap month incorrectly for that year. This "Chinese Y2033" issue was what finally motivated me to write this paper.

In this article I first explain the rules for the Chinese calendar, leading to a discussion of the Y2033 problem. I then discuss some other mathematical issues in the Chinese calendar. Many Chinese astronomers claim that there can be no leap month after the 12th or 1st month ([48]). This is true in the sense that it has not happened since the last calendar reform in 1645, and that it will not happen in the 21st century. But because of the precession of the equinoxes (Section 2) it is clear that in the future there will be many such leap months. In 2262 there will be a leap month after the 1st month, and in 3358 there will be a leap month after the 12th. Given the difficulty in making accurate astronomical predictions more than 100 years ahead, these computations must obviously be taken with a grain of salt. I also believe that there was an error in the computations for 1651, and that there should have been a leap month after the 1st month that year. Instead the leap month was added after the 2nd month.

There are many different statements about what are the possible dates for Chinese New Year. In the 1000 years between 1645 (the last calendar reform) and year 2644, Chinese New Year will always fall between January 21 and February 21.

I know of at least three commonly stated, but not always correct, rules for determining the date for Chinese New Year.

1. *Chinese New Year falls on the day of the second new Moon after the December solstice on approximately December 22.* This fails whenever there's a leap month after the 11th or 12th month. In 2033 it will fail for the first time since 1645 (the last calendar reform).

2. *Chinese New Year falls on the day of the new Moon closest to the jiéqì (*节气 [節氣]*) beginning of spring* (立春, lìchūn) *on approximately February 4.* This rule failed in 1985 and will fail again in 2015.

3. *Chinese New Year falls on the day of the first new Moon after the zhōngqì (*中气 [中氣]*) dàhán* (大寒) *on approximately January 20.* This rule failed in 1985 and will fail again in 2053.

At the end I give an outline of certain aspects of the history of the Chinese calendar related to this paper.

For the computations in this paper I used the Mathematica version of the code from the book by Dershowitz and Reingold ([11]). The conversion from Lisp to Mathematica was done by Robert C. McNally. Their astronomical functions are based on the book by Meeus ([36]). Additional functions are in my Mathematica package ChineseCalendar ([1]). I also highly recommend Chinese Calendrics Software ([20]).

I would like to thank a number of people who have inspired me over the years. My father, Edvard Aslaksen, for making me interested in astronomy by first pointing out the equation of time to me, Professor Wu-Yi Hsiang, my Ph.D. advisor in the Department of Mathematics at UC Berkeley, for teaching me that "first you search, then you search again, and then you research", and Professor Frederic E. Wakeman Jr. of the Department of History at UC Berkeley whose lectures on Chinese history were models in terms of teaching, scholarship and stand-up comedy.

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2 A quick course in astronomy

2.1 Solstices and equinoxes

In order to explain how the Chinese calendar works, we must start by recalling some basic facts from astronomy. For the purpose of reference, I will go into more detail than is strictly necessary, so the reader may skip parts of this section. For excellent introductions to spherical astronomy, see the books by Evans ([16]), Kaler ([24]) and Rogers ([42]). For technical details, I rely on the books by Meeus ([35, 36]) and the Explanatory Supplement to the Astronomical Almanac ([12]).

The Earth revolves counterclockwise (when viewed from the north celestial pole) around the Sun in an elliptical orbit (Figure 1). The plane of the orbit is called the *ecliptic* plane. The word ecliptic is derived from the fact that eclipses can only occur when the Moon crosses this plane. The Earth rotates counterclockwise around an axis that is tilted approximately 23.5 degrees to the normal to the ecliptic plane. Notice how astronomers make a distinction between *revolving* and *rotating*. An object rotates around an axis that passes through it, but it revolves around some outside object.

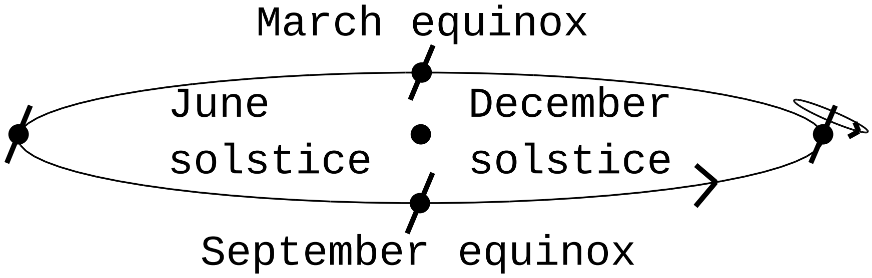

Figure 1: The ecliptic plane

Early astronomers realized that the motion of the Sun along the ecliptic was not uniform. This is a consequence of Kepler's Second Law, which says that the planets sweep out equal areas in equal time (Figure 2). This means that the Earth moves faster along the orbit near *perihelion*, the point on the orbit where the Earth is closest to the Sun.

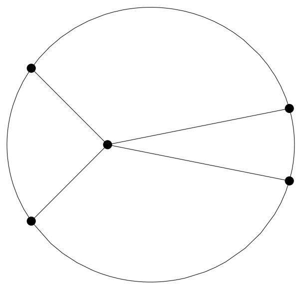

Figure 2: Kepler's Second Law

When the Earth's axis tilts towards the Sun, there is summer in Beijing. At the June and December *solstices*, also called the summer and winter solstices, the projection of the Earth's axis onto the ecliptic plane points directly towards the Sun (Figure 3). At the March and September *equinoxes*, also called the vernal (spring) and autumnal equinoxes, the radial line from the Sun to the Earth is perpendicular to the Earth's axis. These four points are called *seasonal markers*.

The above definition is of course not the way people in ancient civilizations determined the seasonal markers. A simple way was to look at how the rising position of the Sun changes over the course of the year. The Sun rises due east at the equinoxes, at which time day and night are equally long. The word equinox is derived from a Latin word meaning equal night. Strictly speaking, the day is a bit longer at the time of the equinox, since sunrise is the time when the top of

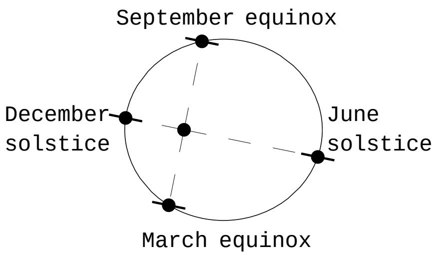

Figure 3: Solstices and equinoxes

the Sun reaches the horizon, while sunset is the time when the top of the Sun goes below the horizon. In addition, refraction bends the image of the Sun upwards near the horizon. After the March equinox, the rising position of Sun moves north until the Sun reaches its northernmost rising position at the June solstice. Here the Sun seems to stand still, and the word solstice is derived from the Latin word solstitium, which means standing Sun. The Sun is now in the zenith over the Tropic of Cancer. The word tropic is derived from the Greek word for turning, because the Sun now turns and starts moving south, rising due east at the September equinox before it reaches its southernmost rising position at the December solstice, at which time it is in the zenith over the Tropic of Capricorn. At the time when the terms Tropic of Cancer and Tropic of Capricorn were coined, the solstices were in the zodiac constellations of Cancer and Capricorn. Because of precession (Section 2.2) they have since moved. Figure 4 shows the path of the Sun in the sky for observers in Beijing and Singapore.

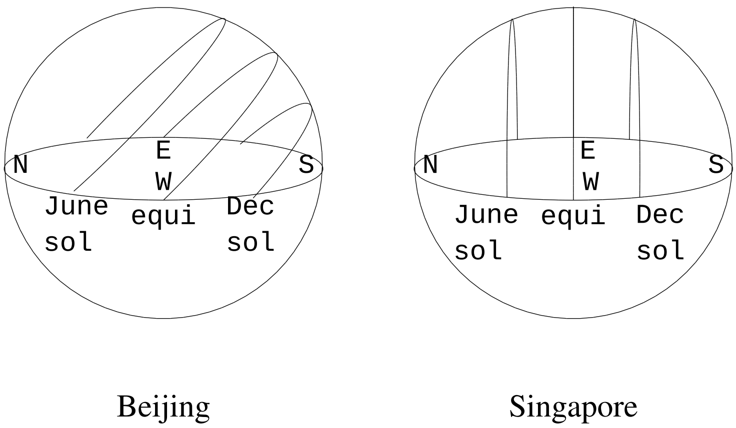

Figure 4: The daily path of the Sun in the sky

A more accurate method is to study the shadow of a vertical pole, called a gnomon. At the equinoxes, the path of the shadow cast by a gnomon is a straight line. The rest of the year, the path is a hyperbola. In order to understand this, think of how the line from the Sun to the tip of the gnomon forms a cone in the course of the day. The hyperbola is the conic section obtained by intersecting this cone with the plane corresponding to the Earth's surface. At the equinoxes, the cone degenerates to a cone with vertex angle equal to 180°, i.e., a plane, so the intersection is a line. On the northern hemisphere, the noon shadow is shortest at the June solstice and longest at the December solstice. Chinese astronomers were experts at using the gnomon. The famous mathematician Guō Shǒujìng (郭守敬, 1231–1316) had a 13 meter gnomon built in 1276!

A common mistake is to think that the projection of the Earth's axis is tangential to the orbit at the equinoxes (Figure 3). It is an easy exercise in analytic geometry to see that the radial line from a focus is never perpendicular to the tangent line.

2.2 Precession of the equinoxes

Another common error is to believe that the seasonal markers coincide with the vertices of the elliptic orbit, e.g., to think that the December solstice coincides with perihelion, the point on the orbit that is closest to the Sun. This was the case around 1246, but is no longer true. In order to understand this, we must explain a phenomenon called *precession*, or more formally, precession of the equinoxes. Around 150 BCE, the Greek astronomer Hipparchos (Hipparchus in Latin) discovered that older star records did not match his observations. He realized that the position of the March equinox had drifted backwards along the ecliptic. The Earth's axis revolves around in a circle with a period of about 25,800 years (Figure 5). This makes the March equinox move clockwise by 50" with respect to the stars each year. The reason for this, first explained by Newton, is that the Sun's gravitation tries to straighten the earth. In China, precession was discovered by Yú Xǐ (虞喜, fl. 307–338) around 320, but it was first implemented in a calendar by Zǔ Chōngzhī (祖冲之 [祖衝之], 429–500) in his Dàmíng Calendar (大明) in 462 ([5]).

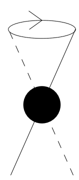

Figure 5: Precession

Some early astronomers measured the time from one December solstice to the next and called it the *tropical year* because it measured the return of the Sun to the same tropic. In Western astronomy it became customary to measure the time between two March equinoxes and call this the tropical year. We will see below that these are not the same ([33])!

Most people think of a year as the time it takes the Earth to complete one revolution around the orbit. This is not true! The time it takes for the Earth to complete one revolution with respect to the stars is 365.25636 days ([11]) and is called the *sidereal year*. This is about 20 minutes longer than the tropical year. The reason for this is that in the course of the year the Earth's axis has rotated clockwise by a small amount, so the axis will be perpendicular to the radial line a bit earlier than a year ago. Hence the Earth covers one orbit minus a small piece. (If you have a hard time visualizing this, you may want to try to think of a solstice year instead.)

Since the speed of the Earth changes according to Kepler's Second Law, the time it takes to cover the extra piece depends on where in the orbit this small piece is. In particular, the time between two December solstices, currently 365.242740 days ([33]), is longer than the time between two March equinoxes, currently 365.242374 days ([33])! The tropical year was traditionally defined to be the mean time from one March equinox to the next. The modern definition is the time it takes the Sun's mean longitude to increase by 360°. It is currently 365.24219 days ([33]). I think that it is very important to be aware of this distinction, but I think it is pedantic to worry too much about it. I will feel free to use the term tropical year for either the value derived from the mean longitude, the March equinox year (used in traditional Western astronomy) or the December solstice year (used in traditional Chinese astronomy). Notice that for all these years, the length is decreasing by about half a second each century ([41]), due to the slowing down of the Earth's rotation.

Since the seasons are tied to the seasonal markers, most calendars try to make their year an approximation of the tropical year, or the seasonal year as it is sometimes called. This is true for the Gregorian and the Chinese calendars, but not for the Indian solar calendars, which use the sidereal year.

In addition to precession there is another factor involved in the behavior of the equinox. The Earth's orbit rotates counterclockwise in the ecliptic plane with a period of about 110,000 years. This means that the orbit rotates in the opposite direction of the precession of the equinoxes. The net effect is that while it takes the March equinox 25,800 years to complete one clockwise revolution with respect to the stars, it only takes about 21,000 years to complete one clockwise revolution with respect to the orbit. The rotation of the orbit has an important consequence. The point where the Earth is closest to the Sun is called *perihelion*. The point where the Earth is farthest from the Sun is called *aphelion*. To say that the equinox rotates clockwise around the orbit is the same as saying that perihelion falls later each year, and that after 21,000 years it drifts through the calendar once. So we can sum up by saying that the solstices and equinoxes move along the orbit with period 21,000 years, but stays fixed in the calendar, while perihelion stays fixed in the orbit, but progresses through the calendar with period 21,000 years.

It follows from this that at the present time, the Earth moves fastest during the winter in Beijing, but in 10,500 years, it will move fastest during the summer.

As always in astronomy, there are some complicating factors! First of all, the Moon deforms the orbit of the Earth. This is because the elliptical orbit is the orbit of the Earth-Moon barycenter (center of mass), while the orbit of the Earth itself is more complex. Since the Earth's orbit is almost circular, even small deformations can change the position of perihelion noticeably. The net result is that perihelion might be up to 32 hours off the expected time ([35]). The time of the solstices and equinoxes, however, are not so sensitive to these deformations.

Secondly, the Gregorian calendar is not a perfect approximation of the tropical year. The insertion of leap days and the fact that the Gregorian year is somewhat longer than the tropical year shifts the time for the seasonal markers and perihelion. Table 1 gives the extreme dates for the seasonal markers and perihelion for the period 1980 to 2020 ([34]).

	Earliest	Latest
Perihelion	January 1, 22h (1989)	January 5, 8h (2020)
March equinox	March 20, 4h (2020)	March 21, 5h (1983)
June solstice	June 20, 22h (2020)	June 21, 23h (1983)
September equinox	September 22, 14h (2020)	September 23, 15h (1983)
December solstice	December 21, 10h (2020)	December 22, 11h (1983)

Table 1: Extreme dates for the seasonal markers and perihelion between 1980 and 2020

2.3 The celestial sphere

Ancient civilizations used geocentric models, and from that point of view, the Sun moves along a great circle on the celestial sphere called the ecliptic (Figure 6). Since you cannot see the Sun and the stars at the same time, it is not immediately obvious how the Sun moves among the stars. But by noticing which stars become visible right after sunset near the spot where the Sun crossed the horizon (heliacal setting) or are visible right before the Sun rises (heliacal rising), it is possible to chart the course of the Sun across the celestial sphere. The equinoxes are the points where the ecliptic intersects the celestial equator, and the solstices are the points where the ecliptic and the celestial equator are farthest apart.

It is important to understand that for the purpose of calendar theory, it does not matter whether we take a heliocentric or geocentric point of view. What matters is the quality of our tables. In fact, the first tables based on the Copernican system were worse than the old tables based on the Ptolemaic system ([16])!

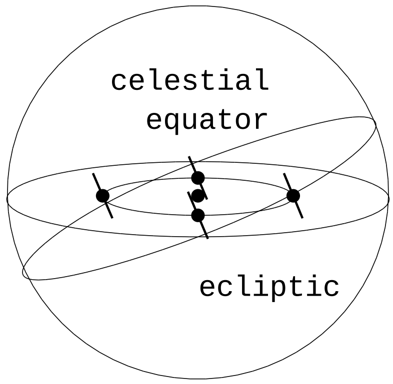

Figure 6: The celestial sphere

The motion of the Moon is very complex. The *synodic month* (or lunation) is the mean time from one new Moon (conjunction) to the next. The word synodic comes from the Greek word synodos or meeting, referring to the Moon's conjunction with the Sun. Between 1000 BCE and 4000 CE it ranges from 29 days 6 hours and 26 minutes (29.27 days) to 29 days 20 hours 6 minutes (29.84 days) with a mean of 29 days 12 hours 44 minutes 3 seconds (29.530588853 days) ([30, 36]).

3 Basic calendrical concepts

3.1 The year

If we consider a *lunar year* consisting of 12 mean lunar months, the length equals 354.36707 days ([12]), which is about 11 days shorter than a tropical year of 365.242374 days. This difference between the lunar year and the solar year was a fundamental problem for the ancients. They tried to overcome the problem by trying to find some longer resonance cycle. Early astronomers discovered that 235 mean lunar months is almost the same as 19 tropical years. In fact, 235 mean lunar months equals 6939.6884 days, while 19 tropical years equals 6939.6018 days. The difference is only about two hours, which accumulates to an error of one day in about 220 years. This is called the *Metonic cycle* after the Greek astronomer Meton who used it in 432 BCE, but it was known to the Babylonians by around 500 BCE and to the Chinese around 600 BCE ([4, 22]). In China it was called the zhāng (章) cycle. Unfortunately, many ancient astronomers were so convinced of the harmony of the heavens that they assumed that this wonderful relation was exact, so for a long time this cycle was hardwired into the Chinese calendar. The Metonic cycle is used in the Jewish calendar, in the computation of Easter, and was used in the Chinese calendar before 104 BCE.

There are several ways to classify calendars. A *lunar* calendar is a calendar that ignores the Sun and the tropical year (and hence the seasons) but tries to follow the Moon and the synodic month. An example of this is the Muslim calendar. The Muslim calendar is based on first visibility of the crescent Moon. Since 12 lunar months is about 11 days shorter than the tropical year, the Islamic holidays regress through the seasons.

Some sources describe an arithmetical (tabular) Islamic calendar. It is sometimes used for approximate conversions for civil purposes, but is not used for religious purposes by Sunnis or Twelver (Ithna Asharia) Shi‘ites. However, it is common among Sevener (Isma‘ili) Shi‘ites, including the Bohras (Musta’lis) and Nizaris (Isma‘ili Khojas, Aga Khanis). It seems to have been designed to be closer to new Moon than to the first visibility of the lunar crescent, so it often runs a day or two ahead of the regular Islamic calendars. There are currently about one million Bohras and about 15 millions Nizaris, compared to over a billion Sunnis and close to a hundred million Twelver Shi‘ites. Both of these groups are today primarily Indian Muslim groups, but they trace their history from the Fatimid Caliphate that ruled Egypt from about 970 to 1171. The calendar was put into practice by Imam al-Hakim (985–1021) and is therefore sometimes referred to as the Fatimid or misr (Egyptian) calendar. The calendar is sometimes attributed to the famous astronomer Al-Battani (850–929) and an alternative version to Ulugh Beg (1393–1449). It is also sometimes referred to as hisabi. It is possibly also used by the Qadianis (Ahmadiyyas), but they also seem to use a solar calendar, and they are not considered Muslims by other Muslims.

The average lunar year is about 354 11/30 days, so you get a reasonable lunar calendar by using a cycle of 11 leap years (kasibah) with 355 days in a 30 year cycle. The odd numbered months have 30 days and the even numbered months have 29 days, except in a leap year when the 12th and final month has 30 days. There are several versions for how to space out the 11 leap years. The most common rule is the one followed by the Nizaris Isma‘ili, which uses years 2, 5, 7, 10, 13, 16, 18, 21, 24, 26, 29, but some replace 16 by 15 and the Bohras replace 7 by 8, 18 by 19 and 26 by 27.

A *solar* calendar uses days to approximate the tropical year. An example is the Gregorian calendar. A year consists of 365 days, while leap years have 366 days. Year n is a leap year if n is divisible by 4, but not by 100 or if n is divisible by

400. So 1900 is not a leap year, while 2000 is. The Gregorian calendar has a cycle of 400 years, with average length of the year equal to 365.2425. This is about 27 seconds longer than the current value of the tropical year. There are various estimates for when this will accumulate to an error of one day. Unfortunately, the shortening of the tropical year (Section 2) makes it impossible to predict how the error will accumulate. Some people, however, have suggested that we should adjust the rules by saying that years divisible by 4,000 are not leap years.

Some people argue ([47]) that since in the past the Gregorian calendar was compared to the March equinox year, we should continue to do so. According to this view, the Gregorian calendar is more accurate than people think, because the Gregorian year is closer to the current value of the March equinox year than to the tropical year. My view is that as long as astronomers defined the tropical year to be the March equinox year, it made sense to compare the Gregorian year with the March equinox year, but now that astronomers have redefined the tropical year to be the period of the mean longitude of the Sun, this is what we should compare the calendar with. The modern Gregorian calendar is determined by scientists and not by religious authorities. Leap seconds are inserted by international scientific organizations and not the by the Pope! Some people also claim that the ecclesiastical rule for computing Easter is an essential part of the calendar. However, many Eastern European countries use the Gregorian calendar for civil purposes and the Eastern Orthodox rules for the computation of Easter.

Since the Gregorian year is an approximation to the tropical year, the equinox stays almost constant. The main movement is caused by the insertion of leap days (also called bissextile days). Each normal year is about 6 hours shorter than the tropical year, so the equinox moves a quarter day later in the calendar for three years in a row. The leap year then evens it out, and the equinox jumps about 18 hours earlier. The equinox performs a four step dance: Three small steps backward (later) and one long step forward (earlier). The old Julian calendar kept the rhythm, but the Gregorian calendar misses a beat three times every 400 years. This is illustrated in Figure 7, taken from the vernal equinox entry of Eric Weisstein's World of Astronomy ([52]).

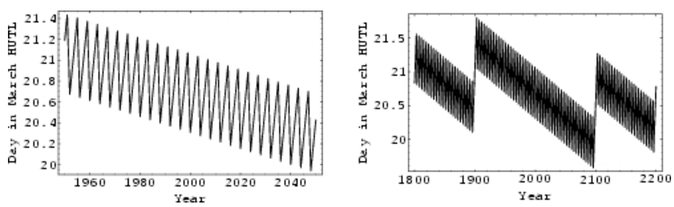

Figure 7: The vernal equinox

As explained in Section 2, the deformation of the orbit caused by the Moon changes the time of the solstices and equinoxes ([35]), as does the fact that the Gregorian year is longer than the tropical year. Table 2 shows the earliest and latest dates for the seasonal markers for the Gregorian cycle between 1800 and 2200. The reason why 1903 and 2096 are the extreme values is because 2000 is a leap year, while 1900 and 2100 are not. As a rule of thumb, I remember the sequence March 20, June 21, September 22, December 21. The traditional dates for the equinoxes and solstices were March 25, June 24, September 24 and December 25.

	Earliest (2096)	Latest (1903)
March equinox	March 19, 14h	March 21, 19h
June solstice	June 20, 7h	June 22, 15h
September equinox	September 21, 23h	September 24, 6h
December solstice	December 20, 21h	December 23, 0h

Table 2: Extreme dates for the seasonal markers between 1800 and 2200

Lunisolar calendars use months to approximate the tropical year. Examples are the Jewish and Chinese calendars. Since 12 months are about 11 days shorter than the tropical year, a leap month (also called intercalary month) is inserted about every third year to keep the calendar in tune with the seasons. The big question is how to do this. A simple method is to just base it on nature. In ancient Israel, the religious leaders would determine the date for Passover each spring by seeing if the roads were dry enough for the pilgrims and if the lambs were ready for slaughter. If not, they would add one more month. An aboriginal tribe in Taiwan would go out to sea with lanterns near the new Moon after the twelfth month. If the migratory flying fish appeared, there would be fish for New Year's dinner. If not, they would wait one month.

A more predictable method is to use the Metonic cycle. Recall from Section 2 that 19 tropical years is almost the same as 235 synodic months. Since $235 = 19 \times 12 + 7$, it follows that we get a fairly good lunisolar calendar if we insert 7 leap months in each 19-year period. The exact rules for this intercalation can be quite tricky, however. This is the method used in the Jewish calendar, and was used in the Chinese calendar before 104 BCE. The modern Chinese calendar is different, in that it uses the motion of the true Moon rather than the mean Moon. I will explain the exact rules for the Chinese calendar in Section 4.

Notice that the Chinese calendar is *not* a lunar calendar! The Chinese name is in fact yīnyáng lì (阴阳历 [陰陽曆]), which simply means lunisolar calendar. Lunisolar calendars are solar calendars that just happen to use the lunar month as the basic unit rather than the solar day.

Since the year in a lunisolar calendar is an approximation to the tropical year, the solstices and equinoxes stay relatively constant. The main movement is caused by the insertion of leap months. Each 12-month year is about 11 days too short, so the solstices and equinoxes move 11 (or 10 or 12) days later. Each 13-month leap year (also called embolismic year) is about 19 days too long, so the solstices and equinoxes jump 19 (or 18 or 20) days earlier. The solstices and equinoxes performs a three step dance: Two small steps backward (later) and one long step forward (earlier). But this dance is a bit off-beat. Two of the seven leap years in each 19-year cycle come after just one normal year, i.e., two years after the previous leap year, so in that case the solstices and equinoxes change into a two step rhythm. Notice also that the steps are much bigger than in the Gregorian four step dance.

There is also another way of classifying calendars. An *arithmetical* calendar is defined by arithmetical rules. Examples are the Gregorian and Jewish calendars. Prediction and conversion between different arithmetical calendars is in principle simple.

An *astronomical* calendar is a calendar that is defined directly in terms of astronomical events. Examples are the Islamic and Chinese calendars. Strictly speaking, there are two kinds of astronomical calendars. The modern (post 1645) Chinese calendar is modular in the sense that the rules are defined in terms of the motion of the true Sun and true Moon, but the description of this motion is considered to be a separate problem and is not specified as part of the calendar. The Jesuit missionaries who carried out the 1645 reform did not have accurate methods for computing this, but since their methods were not hardwired in the calendar, there was no need for further reforms as the computational methods improved. The computational problem is (in principle) separated from the rules for the calendar, and the calendar is (in principle) always correct. Many Indian calendars, however, use traditional formulas for approximating the true motions. These formulas are hardwired into the calendar. In such a *semi-astronomical* calendar, errors are likely to develop, because the traditional methods are not accurate.

For completeness, let me also briefly mention the French Revolutionary calendar. To mathematicians it's interesting to know that Lagrange and Monge were in the committee that formulated the French Revolutionary calendar, while Laplace was in the committee that eventually abolished it ([41]). Its New Year fell on the day of the September equinox. The length of the year depends on which day the next September equinox falls on. For most years, the next September equinox falls on the 366th day of the year. That day is the first day of the new year, so the old year has 365 days. But after 4, or occasionally 5, such years the September equinox falls on the day after the 366th day, and we have a leap year with 366 days. This system has the advantage that it is always in tune with the tropical year, but it is hard to predict which years will be leap years, especially for years when the September equinox happens to occur near midnight. If we wanted a true astronomical solar calendar, we should of course also define the day using the motion of the true Sun rather than the mean Sun. As you can see, this calendar would not be very practical! Still it is an interesting thought experiment, because as we will see, the Chinese calendar is based on related principles.

Table 3 illustrates these classifications. Prediction and conversion involving astronomical calendars is hard, and requires knowledge about the position of the Sun and the Moon.

	Arithmetical	Astronomical
Solar	Gregorian	French Revolutionary
Lunisolar	Jewish	Chinese
Lunar	Civil Muslim	Religious Muslim

Table 3: Classification of calendars

For completeness, I would also like to mention the ancient Icelandic calendar. It consisted of years with 52 weeks, and occasionally a leap year with 53 weeks. It has an interesting history, and the reason why they used weeks instead of months is probably related to the fact that Iceland is almost on the Arctic Circle. That means that in the summer, you will not see the full Moon, and it will be harder to use the Moon for time keeping.

3.2 The month

The Moon is a convenient time keeper. Most ancient calendars started out as observational lunar calendars, with the month starting either with the first visibility of the crescent Moon, or with the full Moon. However, as their astronomical skills developed, most societies switched to starting the month at the computed time of the new Moon. In China, this change seems to have taken place in 621 BCE ([14]). At the moment, only the Islamic calendar starts the month with the first visibility of the lunar crescent. That's why the Muslim month will usually start on the third day of the Jewish or Chinese months. When converting, it is important to remember that the Muslim day starts at sunset, so strictly speaking the Muslim month will usually start on the evening of the second day of the Chinese or Jewish calendar. However, the comparison with the Jewish calendar is complicated by the fact that the Jewish calendar uses mean values for the length of the month, so the first day of the Jewish calendar is not necessarily the exact day of the new Moon.

3.3 The day

One remnant of change from observational lunar calendars to computed lunar calendars is in the convention for the start of the day. The first visibility of the lunar crescent will occur in the West after sunset. For a calendar that starts the month with the first visibility of the lunar crescent it makes sense to start the day at sunset. However, if the month starts with the new Moon, it may be more natural to start the calendar day with sunrise or midnight. Astronomers, however, are nocturnal, and may prefer to start the day at noon. Greenwich Mean Time (GMT) is actually measured from noon whereas Coordinated Universal Time (UTC) is measured from midnight. However, few use the noon measurement and refer to GMT as if it were actually UTC.

Many people are confused by the acronym UTC for Coordinated Universal Time. Some believe that the acronym is based on the French name, but the story is actually even more interesting. In 1970 the Coordinated Universal Time system was devised by an international advisory group of technical experts within the International Telecommunication Union (ITU). The ITU felt it was best to designate a single abbreviation for use in all languages in order to minimize confusion. Since unanimous agreement could not be achieved on using either the English word order, CUT, or the French word order, TUC, the acronym UTC was chosen as a compromise.

When converting between calendars that start the day at different times, we will always match the main daylight parts of the days. So "today" means the Western or Chinese day that started at midnight, the Muslim day that started at sunset last night and the Indian day that started at sunrise this morning.

4 The Chinese calendar

4.1 The 24 Jiéqì

In order to understand the rules for the Chinese calendar, we must first define the 24 jiéqì (节气 [節氣]), called solar terms or solar nodes (or various other names) in English. They are a generalization of the solstices and equinoxes. The seasonal markers cut the ecliptic into four sections of 90° each (Figure 3). The 24 jiéqì cuts the ecliptic into 24 sections of 15° each. The even ones are called major solar terms or zhōngqì (中气 [中氣]), while the odd ones are called minor solar terms or jiéqì. Strictly speaking, the word jiéqì is used in two ways. It can either refer to the 12 odd ones, or it can refer to all 24. Since the English word solar term is not well known, I will use the Chinese word jiéqì since this is familiar to many Chinese. Table 4 gives the names and approximate dates (plus or minus one day).

The reason for the variation in the date of the jiéqì is the same as the reasons for the variation of the dates of the seasonal markers discussed in Section 2. I denote the n'th (odd) jiéqì by Jn, and the n'th zhōngqì by Zn.

The major solar terms Z2, Z5, Z8 and Z11 are simply the Western seasonal

J1	Lìchūn	立春	Beginning of spring	February 4
Z1	Yǔshuǐ	雨水	Rain water	February 19
J2	Jīngzhé	惊蛰 (驚蟄)	Waking of insects	March 6
Z2	Chūnfēn	春分	March equinox	March 21
J3	Qīngmíng	清明	Pure brightness	April 5
Z3	Gǔyǔ	谷雨 (穀雨)	Grain rain	April 20
J4	Lìxià	立夏	Beginning of summer	May 6
Z4	Xiǎomǎn	小满	Grain full	May 21
J5	Mángzhòng	芒种	Grain in ear	June 6
Z5	Xiàzhì	夏至	June solstice	June 22
J6	Xiǎoshǔ	小暑	Slight heat	July 7
Z6	Dàshǔ	大暑	Great heat	July 23
J7	Lìqiū	立秋	Beginning of autumn	August 8
Z7	Chǔshǔ	处暑 (處暑)	Limit of heat	August 23
J8	Báilù	白露	White dew	September 8
Z8	Qiūfēn	秋分	September equinox	September 23
J9	Hánlù	寒露	Cold dew	October 8
Z9	Shuāngjiàng	霜降	Descent of frost	October 24
J10	Lìdōng	立冬	Beginning of winter	November 8
Z10	Xiǎoxuě	小雪	Slight snow	November 22
J11	Dàxuě	大雪	Great snow	December 7
Z11	Dōngzhì	冬至	December solstice	December 22
J12	Xiǎohán	小寒	Slight cold	January 6
Z12	Dàhán	大寒	Great cold	January 20

Table 4: The 24 jiéqì

markers. The minor solar terms J1, J4, J7 and J10 start the Chinese seasons. Notice that in Western astronomy, spring begins at the March equinox, while in Chinese astronomy, spring begins midway between the December solstice and the March equinox. In Western popular culture this convention is often used. The traditional dates for the equinoxes and solstices were March 25, June 24, September 24 and December 25. Shakespeare's A Midsummer Night's Dream takes place on June 23, the eve of Midsummer Day on June 24. To Shakespeare, the June solstice was the middle of summer, not the beginning. Midsummer Day on June 24 is one of the four Quarter Days in the Legal Calendar in the UK. The others are Lady Day (or Annunciation Day) on March 25, Michaelmas on September 29 and Christmas on December 25. These Christian festivals are related to the seasonal markers. Lady Day on March 25 marked the beginning of the year in the UK until 1752. When the UK switched to the Gregorian calendar in 1752 and removed 11 days, they also moved the start of the civil year to January 1, but the start of the financial year was moved to April 5, 11 days after March 25.

The Chinese beginning of season markers also have their analogies in Western culture ([32]). Groundhog Day or Candlemas on February 2 is close to the beginning of spring (lìchūn) on February 4. May Day on May 1 and Walpurgisnacht on April 30 are close to the beginning of summer (lìxià) on May 6. Lammas on August 1 is close to the beginning of autumn (lìqiū) on August 8. Halloween (Hallowmas) on October 31, All Saints' Day on November 1, Guy Fawkes Day on November 5 and Martinmas on November 11 are close to the beginning of winter (lìdōng) on November 8. These Christian holidays are related to the Celtic holidays Imbolg, Beltane, Lughnasa and Samhain ([41]). These holidays are listed in Table 5.

Astronomical	Chinese	Western	Celtic
	Beginning of spring	Groundhog Day, Candlemas	Imbolg
March equinox	Chūnfēn	Lady Day, Annunciation Day	
	Beginning of summer	May Day, Walpurgisnacht	Beltane
June solstice	Xiàzhì	Midsummer Day	
	Beginning of autumn	Lammas	Lughnasa
September equinox	Qiūfēn	Michaelmas	
	Beginning of winter	Halloween, All Saints', Guy Fawkes, Martinmas	Samhain
December solstice	Dōngzhì	Christmas Day	

Table 5: Holidays related to seasonal markers

Two of the jiéqì's are Chinese festivals: Qīngmíng around April 5 and Dōngzhì (December solstice) around December 22. All the other traditional Chinese holidays are lunar in the sense that they will fall on a fixed day in a fixed lunar month in the Chinese calendar, but move in the Western calendar, but these two will stay (almost) fixed in the Western calendar and move in the Chinese calendar. The leap days in the Western calendar cause a variation of a day, while the leap months in the Chinese calendar cause a variation of a month. For example, Qīngmíng will always fall on April 4, 5 or 6, but can fall between the 13th day of the second month to the 17th day of the third month in the Chinese calendar. Many Chinese people are confused about Qīngmíng, and I'm often asked about it at public lectures. One year there was an article in the Straits Times newspaper in Singapore, saying that Qīngmíng falls on the fourth day of the fourth lunar month. I believe that this is an interesting example of mistranslation. It is probably based on a Chinese source that said that Qīngmíng always falls on April 4, which is almost correct. However, in Chinese, the term fourth month (四月) is ambiguous, and can mean either April or the fourth month of the lunar calendar. Since the Chinese source was talking about the Chinese calendar, the translator assumed from the context that it was referring to the fourth Chinese month instead of April. In fact, many Chinese people tend to use the Western names for the Chinese month, and will say April when they mean the fourth lunar month. In this paper I will consistently say fourth month for the Chinese month, and reserve April for the Western month.

We have a similar duality in the ecclesiastical calendar, where Christmas Day and Annunciation Day on March 25 are solar holidays, while all the other holidays are tied to Easter and are therefore lunar.

Because of Kepler's second law, the speed of the (apparent) motion of the Sun across the ecliptic is not constant. This was known to the Chinese astronomers since the 7th century, but it was not until the last calendar reform in 1645 that they started using the true Sun, dìngqì (定气 [定氣]), in their computations of the jiéqì. Before that, they had used the mean Sun, píngqì (平气 [平氣]).

It turns out that it is the zhōngqì's that are most important in the calendar computations. Under the mean Sun system, the length between two zhōngqì's is always about 30.44 days, which is a little bit longer than the lunar months. Hence it is possible to have two new Moons between two zhōngqì's or equivalently, a month without any zhōngqì. Under the true Sun system, the zhōngqì's are closer together during the winter. The time between two zhōngqì's ranges from 29.44 days to 31.44 days (Table 19). So under the modern system it is also possible to get a month with two zhōngqì's.

4.2 The Chinese month

Here are the rules for the Chinese calendar.

Rule 1 *Calculations are based on the meridian* 120° *East.*

Before 1929 the computations were based on the meridian in Beijing, 116°25′ East, but in 1928 China adopted a standard time zone based on 120° East, which is close to the longitude of the republican capital Nanjing, 118°46′ East. Since 1929 the Institute of Astronomy in Nanjing, and since 1949 the Purple Mountain Observatory (紫金山天文台 [紫金山天文臺], Zǐjīnshān Tiānwéntái) outside Nanjing has been responsible for calendrical calculations in China.

This change in base meridian has caused a lot of confusion. We will discuss this more carefully in Section 4.6.

Rule 2 *The Chinese day starts at midnight.*

The Chinese system of 12 double hours start at 11 p.m. This is important in Chinese astrology. Your date and time of birth is determined by the eight characters, bāzì (八字), formed by the pair of cyclical characters for the year, month, day and hour. But for calendrical purposes, the day starts at midnight.

Rule 3 *The day on which a new Moon occurs is the first day of the new month.*

Notice that the new month takes the whole day, no matter what time of the day conjunction occurs. So even if the new Moon takes place late in the evening, the whole day is considered to be part of the new month, and if a zhōngqì occurred in the early morning, it is considered as having fallen in the new month, even though it may have occurred almost 24 hours before the new Moon.

The length of the months are determined astronomically (Table 6). Suppose a month is 29.5 days, and starts with a new Moon at 13h on May 1. The next new Moon then takes place at 1h on May 31, so the month has 30 days. But if the new Moon occurred at 1h on May 1, then the next new Moon would be at 13h on May 30, so the new month would start one day earlier, and we would only get 29 days in the month.

New Moon	Next new Moon	Length
May 1, 13h	May 31, 1h	30 days
May 1, 1h	May 30, 13h	29 days

Table 6: Determining the length of the months

In the Gregorian calendar all the months (except for February) have the same number of days in different years. This is not the case for the Chinese calendar. A month may have 29 or 30 days in different years. Since the mean synodic month is 29.53 days, a little over half the months are big months, dàyuè (大月), with

30 days and a little less than half the months are small months, xiǎoyuè (小月), with 29 days. From a naive point of view, we would expect them to more or less alternate, with occasionally two long months, liándà (连大 [連大]), in a row. This was the method until the start of the Táng Dynasty (唐朝, 618–907) in 619, when the mean Moon, píngshuò (平朔), was abandoned in favor of the true Moon, dìngshuò (定朔). The motion of the true Moon is highly irregular, and it turns out that it is possible to have up to four big months or three small months in a row. For an example of four big months in a row, consider the sequence of new Moons given in Table 7 ([48]).

New Moon	Length
1990 Oct 18, 23h 36m	29d 17h 29m
1990 Nov 17, 17h 5m	29d 19h 17m
1990 Dec 17, 12h 22m	29d 19h 28m
1991 Jan 16, 7h 50m	29d 17h 42m
1991 Feb 15, 1h 32m	

Table 7: Four big months in a row

The next string of three short in a row will start in June 2089. In fact the occurrence of strings of four long or three short is very irregular. I have computed all such strings between 1645 and 2644, and with one exception, all of them occur in strings of such strings, with sometimes long gaps between them. Table 8 shows all such strings between 1646 and 2496. The strings of short months all occur during the summer and the strings of long months occur during the winter. This is because the Earth is moving faster in the winter, which tends to make the lunations longer. Many of the strings are about 9 years apart. This is related the fact that the Moon's perigee has a period of 8.85 years. I will write more about this in another paper.

The Mid-Autumn Festival is celebrated on the 15th of the month in order to make it coincide with the full Moon. But will the full moon really occur on the 15th? The motion of the Moon is very complex, and it turns out that the full Moon can fall on the 14th, 15th, 16th or 17th. Table 9 shows the day of the full Moon between 1984 and 2049. We see that the most common day is in fact the 16th day.

For examples of this, the full Moon on October 8, 1995, fell on the 14th day of the Chinese month, while the full Moon on April 5, 1996, fell on the 17th day of the Chinese month.

3 short				1735	1743	1744	1752	1753	1760
4 long	1646	1700	1708						

3 short	1761	1762	1769	1770	1788	1797	1805	1806	1814
4 long									

3 short	1822								2089
4 long		1921	1929	1983	1991	2037	2045	2053	

3 short	2097	2098		2133	2142	2143	2150	2151	2152
4 long			2107						

3 short	2158	2159	2160	2167	2168	2176	2177		
4 long								2328	2336

3 short						2487	2488	2495	2496
4 long	2382	2390	2398	2444	2452				

Table 8: Strings of short and long months between 1646 and 2496

14th day	6
15th day	306
16th day	380
17th day	124

Table 9: Day of the full Moon between 1984 and 2049

4.3 The Chinese year

It is important to understand that the Chinese calendar is a combination of two calendars, a solar calendar and a lunisolar calendar. The solar calendar starts at the December solstice and follows the 24 jiéqì. This is traditionally called the farmer's calendar (农历 [農曆]). The lunisolar calendar starts at Chinese New Year and consists of 12 or 13 months. This is what most people think of as the Chinese calendar, but unfortunately the term farmer's calendar has come to include the lunisolar calendar. The Chinese solar calendar follows the tropical year closely, so it is perfect for farming purposes, but the lunisolar calendar is not at all suitable for farmers.

There are therefore two different years in the Chinese calendar, the suì (岁 [歲]) and the nián (年). A *suì* is the solar year from one December solstice to the next. This is similar to the tropical year (except that in Western astronomy the tropical year was traditionally measured from one March equinox to the next). A *nián* is the Chinese year from one Chinese New Year to the next. Since a Chinese year can contain 12 or 13 lunar months, and they can each have 29 or 30 days, the length of a nián can be 353, 354 or 355 in case of a normal year and 383, 384 or 385 days in case of a leap year. There are many conflicting figures for the number of days in a Chinese year. Tang ([48]) does not include 385, but there will be 385 days in 2006. Table 10 gives the distribution of the length of the years between 1911 and 2110. There were 354 days in 1965 and 385 in 1925, 1944 and 2006.

353 days	354 days	355 days	383 days	384 days	385 days
1	84	41	5	66	3

Table 10: The length of Chinese years between 1911 and 2110

In modern Chinese, the word suì is only used when talking about a person's age. Traditionally, Chinese people count their age from the December solstice, but in some parts of China they instead count from Chinese New Year, the seventh day of the new year (人日) or from lìchūn. Using the word suì when talking about a person's age is probably related to this custom.

The suì can either be thought of as the exact time between two consecutive December solstices, in which case the average value is 365.242740 days, or we can think of the solar year as starting on the day of the December solstice and ending on the day before the next December solstice. In the latter case, the suì will always contain a whole number of days. The traditional Chinese way is to think of it as the exact value, but I will use whichever is convenient.

Some astrological sources also use a third year running from one beginning of spring (lìchūn) to the next, and claim that your Chinese zodiac animal (生肖屬相, shēngxiào shǔxiāng) should be based on this. In 1960, Chinese New Year fell on

January 28 while the beginning of spring fell on February 5. If you were born on February 1, you would not be considered a rat, but a pig!

This lìchūn to lìchūn year is sometimes also called a suì. However, I reserve that term for the winter solstice to winter solstice year because that is used in the rules for leap months and because of the significance of the winter solstice in Chinese astronomy. However, the main point is that regardless of definition, the suì has fixed length while the nián has variable length.

Just as we think of the Gregorian year as an approximation to the tropical year, we can think of the nián as an approximation to the suì. This again shows that the Chinese calendar is in a sense really a solar calendar that just uses lunar months rather than solar days as the basic unit.

Let me clarify some terminology. When I talk about the Chinese year 2033, I mean the nián from Chinese New Year 2033 to Chinese New Year 2034. The problem with this convention is that dates in the 11th or 12th months may fall in the following Gregorian year. For example, the 12th month of the Chinese year 2033 starts in January 2034. The suì 2033 is the suì from the December solstice in 2032 to the December solstice in 2033.

The suì can be divided into 12 whole months and about 11 days, or 11 whole months and about 40 days. Table 11 gives two examples.

365 days		
5 days	354 days (12 months)	6 days
13 days	325 days (11 months)	27 days

Table 11: Determining the number of months in a suì

When I say that 2033 is a leap year, it means that the nián 2033 contains 13 months. I will now define a leap suì.

Rule 4 *The December solstice falls in month 11. A suì is a leap suì if there are 12 complete months between the two 11th months at the beginning and end of the suì.*

If there is a new Moon on the day after the December solstice or within about 11 days, the suì is a leap suì. If there is a new Moon on the same day as the December solstice or the first new Moon after the December solstice is more than about 12 days later, it is a normal year. Notice that the leap year test applies to suì's and not to nián's. This again illustrates the fact that the Chinese calendar is primarily solar.

We will see later that in 2033, the leap month follows the 11th month. One of the rules is that the December solstice always falls in the 11th month. Hence 2033 is a leap year but *not* a leap suì, while 2034 is a leap suì but not a leap year.

If we consider the first December solstice and the first 11th month as part of the suì, but not the second December solstice and 11th month, then a leap suì contains 13 months and 12 zhōngqì's. Hence there must be at least one month without a zhōngqì. Notice that in extreme cases (Section 4.4), there may also be a month with two zhōngqì's, and hence two months without a zhōngqì.

Rule 5 *In a leap suì, the first month that does not contain a zhōngqì is the leap month, rùnyuè* (闰月)*. The leap month takes the same number as the previous month.*

Notice that *any* month can have a leap month. Provided there are no months with two zhōngqì's there is exactly one zhōngqì in each non-leap month, and the number of the month is the same as the number of the zhōngqì. Months with two zhōngqì's could only happen after the Jesuits reformed the calendar in 1645, so the fact that each month gets a number from the corresponding zhōngqì is probably part of reason for this rule.

Let me try to illustrate this idea. I run a lot, and on one of my training runs I run up a very gentle hill with small steps that are far apart. The distance between the steps is a little bit more than the length of my running stride. On most strides I climb to the next step, but once in a while, I land near the edge, and I have to take a recovery stride on the same level. If you think of the steps as the zhōngqì's, and my stride as the lunar months, you get a nice analogy with the leap month rule in the Chinese calendar. Another way to think of it, is to say that whenever the lunar months have gotten too far ahead of the zhōngqì's, they need to take a pause (leap month) to let the zhōngqì's catch up.

In recent years, some people have started saying that when a Gregorian calendar month contains two full Moons, then the second is called a blue Moon. This term has an interesting history ([39]). This concept is somewhat similar to the system of Chinese leap months.

The date of Chinese New Year follows from these rules. For more details see Section 4.8.

4.4 Why is 2033 an exceptional year?

Let us start by looking at the times for the zhōngqì's and new Moons at the end of 2033. This is given in Table 12. I denote the n'th month (or the n'th new Moon) by Mn, and I denote the new Moon after Mn by Mn+ and the new Moon after that by Mn++. The reason is that before I have compared with the zhōngqì's, I cannot tell whether any of them are leap months or not. I denote a leap month after Mn by Mn-leap.

M7:	2033 7 26 16h 11m	Z7:	2033 8 23 3h 0m
M8:	2033 8 25 5h 38m	Z8:	2033 9 23 0h 50m
M9:	2033 9 23 21h 38m	Z9:	2033 10 23 10h 26m
M10:	2033 10 23 15h 27m	Z10:	2033 11 22 8h 14m
M11:	2033 11 22 9h 38m	Z11:	2033 12 21 21h 44m
M11+:	2033 12 22 2h 45m	Z12:	2034 1 20 8h 25m
M12:	2034 1 20 18h 0m		

Table 12: Times for the zhōngqì's and new Moons during the winter of 2033/34

It can be seen from Table 4 that the zhōngqì's all occur between the 19th and 23rd of the month. The date of the new Moon, however, more or less regresses through the Gregorian month. If you write out the Gregorian calendar with the months as columns, and mark the new Moons and the zhōngqì's, you see that the zhōngqì's form a more or less horizontal line, while the date of the new Moon climbs upwards until it reaches the top and jumps to the bottom and starts climbing again. Leap months occur when the new Moon curve crosses the zhōngqì curve. Most of the time you get a clean crossing, but sometimes the curves might get intertwined is complex ways. In 1998 (Table 13) the zhōngqì's fell before the new Moon until June, in July they coincided, and from August on the zhōngqì's fell after the new Moon. This clean crossing gave an ordinary leap year. In 2033 (Table 14), however, the zhōngqì's fall before the new Moon until August, and for 7 months between September and March they either coincide, or the zhōngqì fall earlier again. Not until April do the zhōngqì's fall after the new Moon.

	June	July	August
19			
20			
21	Z5		
22			M7
23		Z6 M6	Z7
24	M5-leap		

Table 13: Position of the zhōngqì's and new Moons in 1998

The distribution of the zhōngqì's for the different months during the winter of 2033/34 is given in Table 15.

We see that the 9th month takes Z8, the 10th month takes Z9, and the 11th month takes Z10. But the 11th month also holds on to the December solstice, Z11. The fact that the 8th month does not have a zhōngqì is compensated for by the fact that the 11th month has two. Hence the suì 2033 has only 11 complete

	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr
18							Z1		
19							M1		M3
20						Z12/M12		Z2/M2	Z3
21					Z11				
22				Z10/M11	M11-leap				
23	Z7	Z8/M9	Z9/M10						
24									
25	M8								

Table 14: Position of the zhōngqì's and new Moons in 2033/34

Month	Number of zhōngqì's
2033 M7	1
2033 M8	0
2033 M9	1
2033 M10	1
2033 M11	2
2033 M11-leap	0
2033 M12	2
2034 M1	0
2034 M2	1

Table 15: Distribution of zhōngqì's during the winter of 2033/34

months, while the suì 2034 has 12 complete months. In other words, suì 2033 is not a leap suì, while suì 2034 is a leap suì. It follows that the month after the 7th month is not a leap month, because there's no room for a leap month in the suì. The 8th month is a *fake* leap month, in the sense that it does not contain any zhōngqì, but is not a leap month. This was an error in all Chinese calendars up until the early 1990's.

Year	Leap year	Leap suì	Leap month	Month with 2 zhōngqì's	Fake leap month
1832	Yes	Yes	9-leap	11	
1833	No	No			1
1851	Yes	Yes	8-leap	12	
1852	No	No			2
1870	Yes	Yes	10-leap	11	12
1984	Yes	Yes	10-leap	11	
1985	No	No			1
2033	Yes	No	11-leap	11, 12	8
2034	No	Yes			1

Table 16: Fake leap months

It is clear that fake leap months are closely related to months with two zhōngqì's. Table 16 shows all such months between 1800 and 2100. Notice that 1832, 1851, 1870 and 1984 are both leap years and leap suì's. This is related to the fact that the December solstice is early in the 11th month. 2033 is unique in that the December solstice is the *second* zhōngqì in the 11th month. Since there is a leap suì if M11+ falls within about 11 days of Z11, we see that this is the reason why 2034 is a leap suì while 1833, 1852, 1871 and 1985 are not. The fake leap month in 2034 is the first fake leap month in a leap suì since 1645. The next will occur in 2129. It also follows that Chinese New Year is the *third* new Moon after the December solstice in 2034. Notice also that 2033 contains *two* months with two zhōngqì's. It is interesting to observe that in the Indian calendar, a fake leap month is counted as a leap month, but when a month has two zhōngqì's they skip a month!

A month with two zhōngqì's will of course have three jiéqì's . Sometimes there are months with three jiéqì's where one is a zhōngqì and two are odd jiéqì's. This happened in the 10th month in 1999. They are not so interesting since they do not affect the leap months.

4.5 What is a blind year?

A year is said to have double spring (双春 [雙春], shuāngchūn), if it contains a J1, beginning of spring (lìchūn) at both its beginning and end. This happens if and only if the year is a leap year. Since a double spring year must have at least 367 days, a year must be a leap year in order to have double spring. Assume that there is a leap year that is not a double spring year. We can assume that it only has a J1 at the end. In that case the first Chinese New Year occurs at least one day after J1, and the second Chinese New Year will then occur at least 19 days after J1, since a leap year must be at least 383 days. But the earliest J1 can be is February 4, and the latest Chinese New Year can be is February 21. This shows that a year has double spring if and only if it is a leap year.

In the same way, a year is said to have double spring, double rain, shuāngchūn shuāngyǔ (双春双雨 [雙春雙雨]), if the nián contains both a J1, beginning of spring (lìchūn), and a Z1, rain water (雨水, Yǔshuǐ), at both its beginning and end. This is considered significant in Chinese astrology. Between 1645 and 2644, this happens only 15 times. It happened in 1699, 1832, 1851 and 1984, and will happen again in 2033 and 2053. We see that these years are almost the same as the exceptional years we have discussed earlier.

Some writers call a year without a J1, beginning of spring, at the beginning but with a beginning of spring at the end a *blind year*, a year without any beginning of spring a *double blind year*, a year with a beginning of spring at the beginning but without a beginning of spring at the end a *bright year* and a year with two beginning of springs a *double bright year*. A double blind year is also sometimes called a *widow year*. This terminology is illustrated in Table 17.

		J1			J1
blind year		CNY	—	—	CNY
double blind		CNY	—	CNY	
bright year	CNY	—	—	CNY	
double bright year	CNY	—	—	—	CNY

Table 17: Years with or without the beginning of spring (J1) (CNY is Chinese New Year)

Notice that double blind years are the same as leap years. Since leap years take the beginning of spring at both ends, the year after a leap year must be a blind year or a double blind year, and the year before a leap year must be a bright year or a double blind year. It follows that there are three possibilities, as described in Table 18.

It follows that in any string of consecutive years, the number of leap years and double blind years is either the same or differ by one. Because of the Metonic

leap year	blind	double blind	leap year
leap year	double blind	bright	leap year
leap year	double blind	leap year	

Table 18: Years linking two leap years

cycle, we would expect a 19 year cycle to contain on the average seven leap years, seven double blind years, and 2.5 blind years and 2.5 bright years.

4.6 Where is the Chinese meridian?

If the new Moon or a zhōngqì happens near midnight, it can be difficult to determine which day it falls on. As explained in Section The Chinese Month, the meridian for the Chinese calendar was changed in 1929 from Beijing (116°25′ East) to standard China time zone (120° East). This change corresponds to about 14 minutes.

If you try to reconstruct the Chinese calendar before 1929, but use the modern meridian, the times of new Moons and zhōngqì's will be about 14 minutes late. In 1805, there was a new Moon on August 24. Using the new meridian, there seventh zhōngqì (Z7) was on 0h 07m on August 24, which would give that month a zhōngqì, so the leap month would come after the seventh month. But at that time they used the old meridian, which would put Z7 about 7 minutes before midnight. The month starting with the new Moon on August 24 would then lose its zhōngqì and become a leap month following the sixth month.

Before 1978, many calendars in Hong Kong and Taiwan were still based on the old imperial calendar from 1908, the year in which the last Qīng emperor ascended the throne. At that time the Chinese astronomers had computed the calendar until 2108, using the Beijing meridian. These computations give times for the new Moons or zhōngqì's that are about 14 minutes early. In particular, they computed that the new Moon that marked the start of the 8th month in 1978 would occur just before midnight at 23h 53m on September 2, 1978, making the 7th month a short month. The astronomers at the Purple Mountain Observatory in Nanjing had computed that the new Moon would occur after midnight at 0h 07m on September 3, 1978, making the 7th month a long month.

The Mid-Autumn Festival is celebrated on the 15th day of the 8th month. Because of this, the Mid-Autumn Festival was celebrated on different days, causing a lot of confusion. After 1978, both Hong Kong and Taiwan have followed the same calendar as China, so at least when it comes to calendars, everybody agrees on a "one-China" policy.

There are similar issues when it comes to the calendars in Japan, Korea and Vietnam. I will just give one example ([51]). Traditionally, the Vietnamese used the Chinese calendar, even though the longitude of Hanoi is 105°55′ East. However, on August 8, 1967, the North Vietnam government approved a lunar calendar specifically compiled for the UT+7 time zone. The following year, the Chinese New Year new Moon occurred on Jan 29 16h 29m. That meant that in the new North Vietnamese calendar, Chinese New Year, known as Tet in Vietnam, would be celebrated on January 29, while in South Vietnam it would be celebrated on January 30. The North Vietnamese Army and the Vietcong guerillas were preparing for what would be known as the Tet Offensive. The instructions were to attack in the early morning of Tet. The units in Da Nang and other Central Vietnamese cities had closer links to North Vietnam and were aware of the calendar change, so they attacked on the morning of January 30, the day after the new Tet, while in Saigon and other cities to the South, everybody was using the traditional calendar, and the attack started on the morning of January 31, the day after the traditional Tet.

4.7 Can any month have a leap month?

Many Chinese astronomers claim that there can be no leap month after the 12th or 1st month ([48]). This is true in the sense that it has not happened since the last calendar reform in 1645, and that it will not happen in the 21st century. Before 1645, the Chinese calendar used the mean Sun, and then all months had leap months with equal probability. Because of precession, the Sun will move faster during the summer in 10,500 years, so by then, there will be lots of winter leap months. But what about our current period?

Z11	29.44	Z12	29.59	Z1	29.97	Z2	30.47
Z3	30.97	Z4	31.34	Z5	31.44	Z6	31.29
Z7	30.89	Z8	30.37	Z9	29.89	Z10	29.55

Table 19: Distance between the zhōngqì's

Table 19 gives the distance between zhōngqì's. In order to have a leap month after the 11th month, M11++ must be on the same day as Z12. Let us assume that our months are 29.53 days long and that the December solstice happens right before midnight with a new Moon happening right after midnight (Table 20). We then get a leap month after month 11.

Z11	M11+	Z12	M11++
-0.01	0.01	29.43	29.54

Table 20: Leap month after the 11th month

In order to have a leap month after the 12th month, Z12 must fall on the day before M12+. The first row in Table 21 shows one attempt that fails because M12+ and Z1 fall on the same day. But by shifting M12 later, I get a leap month after the 12th month (see the second row in Table 21). Table 22 shows a situation that almost gives a leap month after the 1st month. If M1++ fell 0.08 days earlier, we would get a leap month after the 1st month. Given the irregularity of the Moon's motion, this is not impossible.

Z11	M12	Z12	M12+	Z1	M12++
-0.01	0.01	29.43	29.54		
	0.48		30.01	59.02	59.54

Table 21: Leap month after the 12th month

Z11: -0.04	M12: 0.48
Z12: 29.40	M1: 30.01
Z1: 58.99	M1+: 59.54
Z2: 88.96	M1++: 89.07

Table 22: Possible leap month after the 1st month

Based on my computations, I believe that in 2262 there will be a leap month after the 1st month, and that in 3358 there will be a leap month after the 12th. Given the difficulty in making accurate astronomical predictions more than 100 years ahead, these computations must obviously be taken with a grain of salt. I also believe that there was an error in the computations for 1651, and I believe there should have been a leap month after the 1st month that year. Instead the leap month was added after the 2nd month.

Notice that there can only be a leap month between the December solstice and Chinese New Year if there is a new Moon very soon after (but not on the same day as) the December solstice, so the second new Moon would be around January 21 and the third around February 21. So even though a leap month after the 11th or 12th month causes Chinese New Year to fall on the third new Moon after the December solstice, it does not mean that Chinese New Year will fall in March!

I have computed all the leap months between 1645 and 2644. The most common leap month is a 5th leap month. Notice how all the months between the 9th and the 1st very rarely have leap months. The distribution is given in Table 23.

4.8 What is the date of Chinese New Year?

The exact date of Chinese New Year follows from the above rules. In this section we will try to describe more closely the variation in the date of Chinese New Year.

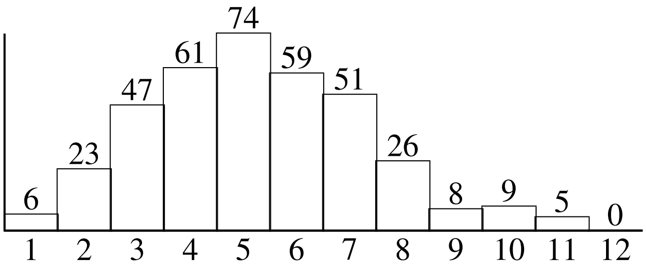

Table 23: Distribution of leap months between 1645 and 2644

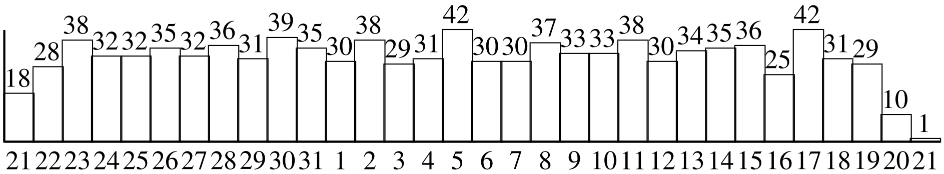

Table 24: Dates of Chinese New Year between 1645 and 2644

According to Tang ([48]), Chinese New Year always falls between January 21 and February 20. This is basically correct. Table 24 shows that the possible dates for of Chinese New Year between 1645 and 2644 are between January 21 and February 21. We see that dates between January 22 and February 19 are common, January 21 and February 20 are rare, and February 21 is extremely rare. Chinese New Year will fall on February 21 in 2319.

The Chinese New Year performs the same off-beat three step dance as the solstices and equinoxes (Section 3.1), but in the opposite direction. It moves earlier by 11 days (or 10 or 12) once or twice, but if a step would take us earlier than January 21, it jumps later by 19 (or 18 or 20) days. This is illustrated in Table 25.

1999	2000	2001	2002	2003	2004	2005	2006	2007
Feb 16	Feb 5	Jan 24	Feb 12	Feb 1	Jan 22	Feb 9	Jan 29	Feb 18
11	12	19	11	10	18	11	20	

Table 25: The movement of the dates of Chinese New Year

There are also three simple and commonly stated, but not always correct rules of thumb that are often given for determining the date for Chinese New Year.

Rule of thumb 1 *Chinese New Year falls on the day of the second new Moon after the December solstice on approximately December 22.*

As explained before, this rule of thumb is correct, provided there is no leap month after the 11th or 12th month. In that case, Chinese New Year falls on the third new Moon after the December solstice. This, however, does not mean that we will be celebrating Chinese New Year in March. It follows from Section 4.7 that there can only be a leap month between the December solstice and Chinese

New Year if there is a new Moon very soon after (but not on the same day as) the December solstice, so the second new Moon will then be around January 21 and the third around February 21. Notice that this rule of thumb only fails when Chinese New Year is exceptionally late.

Rule of thumb 2 *Chinese New Year falls on the day of the new Moon closest to the jiéqì (*节气 [節氣]*) beginning of spring* (立春, lìchūn) *on approximately February 4.*

This rule of thumb is important because it explains why Chinese New Year is called the Spring Festival, Chūnjié (春节 [春節]). In fact, some people say that Chūnjié really refers to lìchūn and not to Chinese New Year! Recall that Chinese astronomers put the seasonal markers in the middle of the seasons, so the start of the seasons fall between the seasonal markers. That's why the Spring Festival can be pretty chilly in Beijing!

This rule of thumb is normally true, but since the beginning of spring falls around February 4 and Chinese New Year falls between January 21 and February 21 it is hard to determine which new Moon is closest if we have a very early or a very late Chinese New Year. The rule of thumb fails 31 times between 1645 and 2644. In 1985 the beginning of spring fell on February 4, 5h 11m, Chinese New Year was on February 20, 2h 42m, while the previous new Moon was on January 21, 10h 28m. The distance between the beginning of spring and the previous new Moon was 13.78 days, while the distance between the beginning of spring and Chinese New Year was 15.90 days, so the rule of thumb failed in 1985. It will fail again in 2015.

At first I thought that it would be possible for this rule of thumb to fail in case of either an early or a late Chinese New Year, but when I tested on a computer, it only failed for late Chinese New Years. To see why, suppose that we have a very early Chinese New Year. In that case Z11 must fall late in the 11th month, but before the day of M12. We must also have Z12 in the month after the 11th month (which is then the 12th month), because if Z12 fell in the second month after the 11th month, we would get a leap month after the 11th month, and this would cause a late Chinese New Year. We must also have Z1 in the 1st month, because if it fell in the 12th month, we would get a leap month after the 12th month, and this would cause a late Chinese New Year. So whenever we have an early Chinese New Year, Z11 must fall before the day of Chinese New Year, and Z1 must fall before the day of M2. Therefore the time between Chinese New Year and J1 is less than the time between Z12 and J1. This is approximately the same as the time between J1 and Z1, which is less than the time between J1 and M1+. Hence it is impossible for the beginning of spring to be closer to the new Moon following Chinese New Year than Chinese New Year itself (assuming that the time from Z12 to J1 is close to the time between J1 and Z1).

Notice also that if Chinese New Year is at the beginning of spring, then the middle of spring should be in the middle of the 2nd month. This explains why the Mid-Autumn Festival is celebrated on the 15th day of the 8th month. It can be shown that the Mid Autumn Festival will fall between September 8 and October 8.

Rule of thumb 3 *Chinese New Year falls on the day of the first new Moon after the zhōngqì (*中气 [中氣]*) dàhán* (大寒) *on approximately January 20.*

This can be expressed as saying that Chinese New Year is the first new Moon after the Sun has entered the Zodiac sign of Aquarius.

Since Z12 normally falls in the 12th month, this rule of thumb holds most of the time, but if we have a very late Chinese New Year, it is possible for Z12 to fall before the day of M12. The rule of thumb fails 23 times between 1645 and 2644. It failed in 1985 and will fail again in 2053.

Notice that all these rules of thumb only fail in case of exceptionally *late* Chinese New Years. These rule of thumbs help explain why the range for Chinese New Year is between January 21 and February 21. This also shows how it is usually possible to determine the approximate date of Chinese New Year if you have a calendar that indicates the phases of the Moon (for the Chinese time zone, UT +8). We have seen above that determining the date of Chinese New Year is hard if there are new Moons near the ends of the period from January 21 to February 21. But if there's a new Moon in that period that is not close to the ends, we can use either of the three rules of thumb to conclude (correctly) that it will be the date of the Chinese New Year. More specifically, if you have a new Moon between January 23 and February 19, you can conclude that it will fall on the date of Chinese New Year. But in 1985 there were new Moons on January 21 and February 20 and in 2319 on January 22 and February 21 and in both cases Chinese New Year fell on the later date.

4.9 The 19-year cycle

Because of the Metonic cycle, there is almost a 19-year cycle in the Chinese calendar. I was born on April 16, 1960. This was the 21st day in the 3rd month in the Chinese calendar. Normally my birthday falls on different days in the Chinese calendar, but my 19th birthday fell on the 20th day in the 3rd month. The same goes for my 38th and 57th birthday. So we see that the 19-year cycle is close but not exact. There are two reasons for this. First of all, the Metonic cycle is off by about two hours. But more importantly, we are now comparing the Chinese calendar not with the tropical year, but with the Gregorian calendar, which is just an approximation to the tropical year. In particular, since 19 is not a multiple of 4, different cycles contain different numbers of leap years.

The 19-year cycle is still apparent when it comes to leap years. Table 26 ([30]) shows the leap years and the leap months between 1805 and 2050. If no leap month number is indicated, it means that the leap month is the same as the previous year in the cycle. The columns contain years in the same 19-year cycle. Our old friends 1832, 1851, 1870 and 1984 (Section 4.4) again appear as exceptional cases, in that they break out of the columns corresponding to their 19-year cycles. Notice how these four exceptional years have leap months in the fall, while the other years in the cycle all have spring leap months. So the jump from 1966 to 1984 is 18 years and 7 months, and the jump from 1984 to 2004 is 19 years and 4 months. I like to think of this as an unstable situation, where the leap month "should" have fallen in the winter, but instead either moves earlier or later. We also notice that 2033 is for once reasonably well behaved, in that it stays in its proper column. But it does insist on a different leap month.

1805–7	1808–5	1811–3		1814–2	1816–6	1819–4	1822–3
1824	1827	1830–4	1832–9		1835	1838	1841
1843	1846	1849	1851–8		1854–7	1857–5	1860
1862–8	1865	1868	1870–10		1873–6	1876	1879
1881–7	1884	1887		1890–2	1892	1895	1898
1900–8	1903	1906		1909	1911	1914	1917–2
1919–7	1922	1925		1928	1930	1933	1936–3
1938	1941–6	1944		1947	1949–7	1952	1955
1957–8	1960	1963		1966–3	1968	1971	1974–4
1976	1979	1982	1984–10		1987–6	1990	1993–3
1995	1998–5	2001		2004–2	2006–7	2009	2012–4
2014–9	2017–6	2020		2023	2025–6	2028	2031–3
2033–11	2036	2039–5		2042	2044–7	2047	2050

Table 26: Leap years and leap months between 1805 and 2050

There is also a similar pattern in the date of Chinese New Year. Table 27 shows the date of Chinese New Year between 1980 and 2017. For most days there's at most a difference of a day (caused by leap years), but notice how 1985 is exceptional. From the discussion of 1984 in Section 4.4, however, this is not surprising.

4.10 The sexagenary cycle

An important aspect of the Chinese calendar is the sexagenary cycle (干支, gānzhī). This is a combination of the 10 heavenly stems (天干, tiāngān), and the 12 earthly branches (地支, dìzhī) ([38]) as listed in Table 28.

1980: Feb 16	1999: Feb 16
1981: Feb 5	2000: Feb 5
1982: Jan 25	2001: Jan 24
1983: Feb 13	2002: Feb 12
1984: Feb 2	2003: Feb 1
1985: Feb 20	2004: Jan 22
1986: Feb 9	2005: Feb 9
1987: Jan 29	2006: Jan 29
1988: Feb 17	2007: Feb 18
1989: Feb 6	2008: Feb 7
1990: Jan 27	2009: Jan 26
1991: Feb 15	2010: Feb 14
1992: Feb 4	2011: Feb 3
1993: Jan 23	2012: Jan 23
1994: Feb 10	2013: Feb 10
1995: Jan 31	2014: Jan 31
1996: Feb 19	2015: Feb 19
1997: Feb 7	2016: Feb 8
1998: Jan 28	2017: Jan 28

Table 27: The date of Chinese New Year between 1980 and 2017

Stems	天干	tiāngān	Element	Branches	地支	dìzhī	Animal
1	甲	jiǎ	Wood	1	子	zǐ	Rat
2	乙	yǐ	Wood	2	丑	chǒu	Ox
3	丙	bǐng	Fire	3	寅	yín	Tiger
4	丁	dīng	Fire	4	卯	mǎo	Rabbit
5	戊	wù	Earth	5	辰	chén	Dragon
6	己	jǐ	Earth	6	巳	sì	Snake
7	庚	gēng	Metal	7	午	wǔ	Horse
8	辛	xīn	Metal	8	未	wèi	Goat
9	壬	rén	Water	9	申	shēn	Monkey
10	癸	guǐ	Water	10	酉	yǒu	Chicken
				11	戌	xū	Dog
				12	亥	hài	Pig

Table 28: The 10 heavenly stems and the 12 earthly branches

To explain how this cycle works, let us denote both the stems and the branches by their numbers. We denote 1 by $(1, 1)$ or (甲,子), 2 by $(2, 2)$ or (乙,丑) and so on up to $(10, 10)$ or (癸,酉). But now we have run out of stems, so we denote 11 by $(1, 11)$ or (甲,戌) and 12 by $(2, 12)$ or (乙,亥). Now we have run out of branches, too, so 13 becomes $(3, 1)$ or (丙,子). We continue in this way through 6 cycles of stems and 5 cycles of branches up to 60, which is $(10, 12)$ or (癸,亥). The next number is then $(1, 1)$ or (甲,子), which starts a new sexagenary cycle.

This cycle is used for keeping track of years, months, days and (double) hours in Chinese astrology. Your date and time of birth is determined by the eight characters (八字) formed by the pair of cyclical characters, or pillar, (柱, zhù), for the year, month, day and hour. The 60-day cycle has been used for keeping track of days since ancient times and go back to at least the 13th century BCE during the Shāng Dynasty (商朝, 1600–1046 BCE). The 60-month cycle is also old. The 60-year cycle was introduced during the Hàn Dynasty ((汉朝 [漢朝, 202 BCE–220) and is related to the orbital period of Jupiter. In modern times, the year cycle is the only one in common use. The branches are often associated with the sequence of 12 animals: rat, ox, tiger, rabbit, dragon, snake, horse, sheep, monkey, rooster, dog, and pig. It is not clear when the branches were associated with the 12 animals, but it seems to have taken place around the time of the Táng Dynasty.

Notice that each branch, or animal, occurs five times in each 60-year cycle. An animal corresponding to an odd number, will meet the stems that correspond to the odd numbers. Year 2000 is the 17th year in the current cycle (see below), so it corresponds to $(7, 5)$ $(17 = 10 + 7 = 12 + 5)$ or (庚, 辰). So we see that it is a metal dragon year, or a golden dragon.

Determining the stem corresponding to a month is easy. The 11th month has branch 1, the 12th month has branch 2, the first month has branch 3 and so on. So the only problem is to keep track of the stem. There are two things to notice here. First of all, this system ignores leap months. The month pillar of a leap month is the same as the month pillar of the previous months! Secondly, why does the first branch correspond to the 11th month and not the first month?

In fact, both of these paradoxes are easy to explain. Since two months can have at most 60 days, the day pillar will still separate two different days. In a sense you can think of a month and its following leap month just as a one long month. And why does the first branch correspond to the 11th month? Because the 11th month contains the winter solstice, which is fundamental to Chinese astronomy!

The hour cycle is similar to the month cycle. The first branch corresponds to the double hour from 11 p.m. to 1 a.m., and so on. Again, we only need to worry about the stem.

According to Ho Peng Yoke ([22]), the year of birth was considered the most important in astrology until the Táng Dynasty, when the month of birth assumed greater importance. Since the Míng Dynasty, the day of birth has become the most important in Chinese eight characters astrology.

The cycle of 12 branches is probably related to the 12 months, while the cycle of 10 stems is probably related to the ancient Chinese 10-day "week", xún (旬). The seven-day week was probably introduced not earlier than the Sòng Dynasty (宋朝, 960–1279) ([38, p. 397]).

4.11 What year is it in the Chinese calendar?

Because of my web page about the Chinese calendar [2], I get a lot of e-mail about the Chinese calendar. I once got an e-mail from a greeting cards company who needed to know which year 2000 would be in the Chinese calendar. The answer is that the Chinese do not have a continuous year count. They started counting from one again with each new emperor. However, some scholars tried to reconstruct ancient Chinese chronology by adding up years of reigns, much the same way some westerners in the past tried to reconstruct Biblical chronology. Some claim that the calendar was invented by the Yellow Emperor, Huángdì (黄帝), in 2637 BCE in the 61st year of his reign. However, others prefer to start the count with the first year of his reign in 2697 BCE. Since these years are 60 years apart, it follows that 1984 was the first year of either the 78th or 79th 60-year cycle. Using this as a starting point, Chinese New Year in 2000 marks the beginning of the Chinese year 4637 or 4697. To give you an example of the level of confusion on this point, in Chapter 3 of Volume III of the translation of the Shoo King (书经 [書經], Shūjīng) by James Legge ([26]), he refers to the current year, 1863, as being in the 76th cycle, implying a starting point of 2697 BCE. However, the book has an appendix on Chinese astronomy, written by John Chalmers, where the starting point is taken to be 2637 BCE! Chalmers actually writes 2636 BCE, but he really means -2636, using the astronomical year count, where 1 BCE is year 0, 2 BCE is -1, etc. This is fairly typical of the level of confusion about the continuous year count in the Chinese calendar, and simply illustrates the fact that the continuous year count is not an integral part of the Chinese calendar, but rather an afterthought. While there are isolated incidents of Chinese scholars who have used it, it only gained popularity with the Jesuit missionaries. Most of the people who use it are Westerners who refuse to believe that it is possible to have a "civilized" society without a linear, continuous year count. That's why I told the greeting cards company to stick with calling it the year of the Dragon!

To add to the confusion, some authors use an epoch of 2698 BCE. I believe this because they want to use a year 0 as the starting point, rather than counting 2697 BCE as year 1, or that they assume that the Yellow Emperor started his year with the Winter solstice of 2698 BCE. In particular, this system was used by Sun Yat-sen (孫逸仙, Sūn Yìxiān or 孫中山, Sūn Zhōngshān, 1866–1925). He and other political activists wanted to use a republican and "modern" year numbering system. This system actually won some acceptance in the overseas Chinese community, and is for example used occasionally in San Francisco's Chinatown. (At least around the time of Chinese New Year!)

However, let me stress again that using an epoch is not the traditional way of counting years in Chinese history. Beginning in the Hàn dynasty, emperors would adopt era name or reign names (年号 [年號], niánhào), which together with the 60-year cycle would fix the year. In the past, the emperors often changed their era names during their reign, but by the time of the Míng and Qīng dynasties, the emperors used the same era name for their whole reign. This system worked well most of the time, but the Kāngxī Emperor (康熙) ruled more than 60 years. He ruled from February 7, 1661 to December 20, 1722. Since Chinese New Year fell on January 30 in 1661, the first year of his reign started on February 18, 1662, and the last year of his reign ended on February 4, 1723. Since both 1662 and 1722 are rényín years, the term Kāngxī rényín (康熙壬寅) is ambiguous. However, this is the only such problem in Chinese history. His grandson, the Qiánlóng Emperor (乾隆) ruled from October 18, 1735, to February 8, 1796. The first year of his rule started on February 12, 1736, but he chose to retire on February 8, 1796, as a filial act in order not to reign longer than his grandfather, the illustrious Kāngxī Emperor. Despite his retirement, however, he retained ultimate power until his death in 1799.

It is well known that the 60-year cycle was introduced during the Hàn Dynasty, so it came as something of a surprise when scholars realized that the 60-day cycle had been in use in the Shāng Dynasty (商朝, 1600–1046 BCE). This shows that the two systems are independent, and there is no point looking for an ancient origin with a (甲,子) day in a (甲,子) month in a (甲,子) year in either 2637 BCE or 2697 BCE. I should also point out, that while Chinese chronology is fairly reliable going back to 841 BCE, and oracle bones with date inscription go back to the 13th century BCE, modern scholars consider the Yellow Emperor to be a mythological figure. So this whole discussion of ancient dates is just a curiosity.

4.12 Additional festivals

As stated in the Introduction, Chinese New Year is a public holiday in China, Taiwan, Hong Kong, Macau, Singapore, South Korea, Indonesia, Malaysia, Brunei, Laos, Vietnam and Mauritius.

The Mid-Autumn Festival (中秋, Zhōngqiū) (Sections 4.2 and 4.8) falls on the 15th day of the 8th month in the Chinese calendar and between September 8 and October 8 in the Western calendar. It is a public holiday in Hong Kong, Macau, Taiwan and South Korea.

Qīngmíng (清明) (Section 4.1) will always fall on April 4, 5 or 6, but can fall between the 13th day of the second month to the 17th day of the third month in the Chinese calendar. It is a public holiday in Hong Kong, Macau, Taiwan and South Korea.

Dōngzhì (冬至) (Section 4.1) is the December solstice on December 21, 22 or 23. It is a public holiday in Macau.

The Dragon Boat Festival (端午, Duānwǔ) falls on the 5th day of the 5th month. It is a public holiday in Hong Kong, Macau, Taiwan and South Korea.

The Double Ninth Festival (重九, Chóngjiǔ) or Chóngyáng (重阳 [重陽]), called Chung Yeung in Cantonese, falls on the 9th day of the 9th month. It is a public holiday in Hong Kong and Macau.

5 A brief history of the Chinese calendar

5.1 Outline of the main calendar reforms

The calendar has always been important in Chinese society. The Chinese emperor based his authority on being the Son of Heaven (天子, tiānzǐ) and the Mandate of Heaven (天命, tiānmìng). It would be a loss of face if the calendar was not in harmony with the heavens. Unfortunately, with a lunar or lunisolar calendar, errors are much more noticeable than with a solar calendar. A solar calendar can be off by a couple of weeks without anybody noticing. The reason why the Catholic church had to reform the Julian calendar was because the rules for computing Easter had fixed the March equinox to be March 21. That meant that Easter was drifting noticeably towards summer. Otherwise, few would have cared about the drift of the March equinox. But with a lunar calendar, an error of even a couple of days is a serious problem. Every peasant would then on the first day of the month see either a waxing crescent or a waning crescent. Why should they pay taxes and serve in the army if the emperor did not know the secrets of the heavens?

For the same reason, prediction of eclipses has always been very important in China. If an eclipse was predicted, but did not occur, it was a sign that Heaven looked favorably upon the emperor. But if an eclipse occurred that the emperor's astronomers had failed to predict, it was taken as a sign that the emperor had lost the "Mandate of Heaven".

If the new Moon or zhōngqì takes place near midnight, even a minor miscomputation can lead to a shift of a whole day for a calendrical event. This means that the demand for accuracy was unlimited.

For these reasons, producing alternative calendars were punishable by death, and there were elaborate ceremonies associated with the promulgation of new calendars. The calendar was an integral part of the Chinese ideological and political system.

Because of the importance the Chinese rulers placed on calendars, they were surprisingly open to incorporate foreign ideas into the making of calendars. The last three main calendar reforms, in the Táng, Yuán and Qīng Dynasties, have all been associated with foreign impulses.

There have been over 100 calendar reforms in China, and much of Chinese astronomy and mathematics was developed in order to improve the calendar. For example, the Chinese Remainder Theorem in number theory was essential in the older, arithmetical calendars ([27]). Unfortunately, the otherwise authoritative work of Needham ([38]) almost completely ignores the topic. (See however the article by Cullen ([7]).) The best reference in English is the article by Chén Jiǔjīn ([5]). I will give a brief outline of some of the main points.

Oracle bones show that the Chinese calendar was in existence already in the 13th century BCE. But at that time, the calendar was uncertain, and some years even had 14 month. Our records of calendars from that period are limited, but from 841 BCE onwards we have detailed knowledge of Chinese dates, based on the "Records of the Grand Historian" (史记 [史記], Shǐjì) by Sīmǎ Qiān (司马迁 [司馬遷], ca. 145-—90 BCE).

Before 621 BCE during the Zhōu Dynasty (周朝, 1027(?)–221 BCE), the Chinese determined the start of the month based on visibility of the crescent Moon ([14]). Before 589 BCE, intercalation varied between six and eight years every 19 years, but after 589 BCE it remained constant at seven intercalations every 19 years ([4, 22]). This shows that the Metonic cycle, named after the Greek astronomer Meton who used it in 432 BCE, but also known to the Babylonians by around 500 BCE, was in fact known to the Chinese about 100 years earlier. In China it was called the zhāng (章) cycle.

In the beginning the leap months were always placed at the end of the year. However, after the Tàichū Calendar (太初) reform in 104 BCE during the Hàn Dynasty ((汉朝 [漢朝, 202 BCE–220), the no zhōngqì (无中气 [無中氣]) rule (Section 4.3) was used for determining leap months, and the month containing the December solstice was fixed to be the 11th month. For details about the calendars during the Hàn Dynasty, see [8, 9, 17, 25, 44, 54].

However, the difference between the no zhōngqì rule and the 19-year cycle is not that big. Consider for example the Sìfēn Calendar (四分) that was used between 85 and 263 during the Later (Eastern) Hàn Dynasty (后汉 [後漢], 25–220). The Metonic cycle was still hardwired into the calendar. The year was taken to be $365\frac{1}{4}$ days and a month was taken as $19/235 \times 365\frac{1}{4} = 29\frac{499}{940}$. The no zhōngqì rule is then almost equivalent to the Metonic cycle. The only problem is the we have to worry about how the new Moons and zhōngqì's fall with respect to midnight. The difference would never be more than one month, though. For details, see [25].

The no zhōngqì rule only became significant in 619 during the Táng Dynasty (唐朝, 618–907) when the calendar switched to following the true Moon. This was inspired by Indian Buddhist astronomers. The Táng reforms culminated with the work of the Buddhist monk Yī Xíng (一行, 683–727) who designed the Dàyǎn Calendar (大衍) in 728.

The next significant reform came in 1280 during the Yuán Dynasty (元朝, 1271–1368). It was inspired by Muslim astronomers, but designed by the famous mathematician Guō Shǒujìng (郭守敬). It was the most accurate calendar in the world at that time.

The last calendar reform came in 1645 during the Qīng Dynasty (清朝, 1644–1911) and was implemented by Jesuit missionaries. It used the true Sun. In a system that uses both mean sun and mean moon, all months have leap months with the same probability, and there are no fake leap months (Section 4.4). In a system that uses mean Sun but true Moon, summer leap months are more likely to occur, but there are no fake leap months. After 1645 leap months are more likely to occur in the summer, and there's also the possibility of fake leap months.

After the 1911 Revolution, the Republican government made the Gregorian calendar the official calendar in 1912. However, the traditional Chinese calendar is still important to Chinese people all over the world when it comes to determining the traditional festivals and holidays.

5.2 The true Moon and the true Sun in Chinese calendars

It took a long time before the true Moon and the true Sun were adopted in the Chinese calendar. We will look more closely at the history of these concepts in Chinese astronomy ([3, 5, 55]).

The irregularity of the Moon's motion was known to Shí Shēn (石申, fourth century BCE) and mentioned by Jiǎ Kuí (贾逵 [賈逵], 30–101) in 85 in the Sìfēn Calendar during the Later (Eastern) Hàn Dynasty (后汉 [後漢], 25–220). However, he did not use this fact in the computations for the calendar. The first time that the irregularity of the Moon's motion was taken into consideration in a calendar was in the Qiánxiàng Calendar (乾象), compiled by Liú Hóng (刘洪 [劉洪], ca. 135–210) in 206 during the Later Hàn Dynasty. However, it was only used in the calculation of eclipses, and the calendar still used the mean Moon to compute the length of the months. Using the true Moon for the length of the months was finally proposed by Hé Chéngtiān (何承天) for the Yuánjiā Calendar (元嘉) in 445 during the Southern and Northern Dynasties (南北朝, 420–589). However, using the true Moon would give strings of three big months and strings of two small months in the calendar, which had never happened before. The authorities objected and Hé Chéngtiān was forced to use the mean Moon to compute the number of days in a month. The first calendar that used the true Moon to determine the length of the month was the Wùyín Calendar (戊寅), compiled by Fù Rénjūn (傅仁均) in 619 during the Táng Dynasty (唐朝, 618–907). However, the occurrence of a string of four big months was seen as too radical a change and they reverted back to the mean Moon in 645. It was only after the Líndé Calendar (麟德) in 665 that the true Moon became commonly accepted for the computation of the length of the months.

The irregularity of the Sun's motion was first discovered by Zhāng Zǐxìn (张子信 [張子信], fl. 520–560) around 560 during the Southern and Northern Dynasties (南北朝, 420–589). Liú Zhuó (刘焯 [劉焯], 544–610) agreed with this theory, and discussed the true Sun in his Huángjí Calendar (皇极 [皇極], 600) during the Suí Dynasty (隋朝, 581–618). He introduced the terminology dìngshuò/píngshuò and dìngqì/píngqì. However, he thought that there would be a sudden change in the speed of the Sun at certain times of the year. The actual movement of the sun was fully understood by the famous astronomer Yī Xíng (一行, 683–727), and he used the theory for computing the time of eclipses in his Dàyǎn Calendar (大衍) from 729. However, the mean Sun was still used for computing the jiéqì until the Shíxiàn Calendar (时宪 [時憲], 1645) during the Qīng Dynasty (清朝, 1644–1911).

One little-known fact is that until the time of the Jesuits, the Chinese divided the circle into $365\frac{1}{4}$ degrees. Combining this with a mean Sun and a tropical year of 365.25 days makes for pleasant calculations! Many scholars argue that much of Chinese science comes from the Babylonians. I am generally quite skeptical of such claims, and dividing the circle into $365\frac{1}{4}$ degrees seems very un-Babylonian!

5.3 The Jesuit missionaries

As explained above, the current Chinese calendar is due to Jesuit missionaries. I would like to give some more details about how this came about ([6, 10, 15, 18, 19, 21, 23, 31, 40, 46, 50, 53, 56]). In 1582, the first Jesuit missionary, the Italian Matteo Ricci (利玛窦 [利瑪竇], Lì Mǎdòu, 1552–1610), came to China. Ricci is often considered the father of Western sinology. He managed to convert a leading Chinese official, Xú Guāngqǐ (徐光启 [徐光啓], 1562–1633). Xú became director of the Board of Rites and was essential to the success of the Jesuits. Among other things, Ricci and Xú translated Euclid into Chinese, and much of modern Chinese mathematical terminology can be traced back to their translation.

At that time, the Chinese calendar was no longer accurate. Positions in the Imperial Astronomical Bureau (欽天監, Qīntiānjiān) had become inherited, and the astronomers no longer understood the principles behind the old calendar. When the capital was moved from Nanjing to Beijing in 1403 during the Míng Dynasty (明朝, 1368–1644), the astronomers did not even know how to adjust their instruments and formulas for the change of latitude! When they made an error of more than half an hour in computing a solar eclipse on December 15, 1610, it caused serious embarrassment. Finally, in 1629 Xu was asked to revise the calendar, and he asked the Chinese and Muslim astronomers in the Bureau and the Jesuits to make predictions for an upcoming solar eclipse on June 21, 1629. The Jesuits had the best prediction, and when Xú was made director of a new Calendrical Bureau (历局 [曆局], Lìjú) on September 1, 1629, he appointed the Swiss Johannes Schreck (Johannes Terrenz/Terrentius, 鄧玉函, Dèng Yùhán, 1576–1630) and the Italian Niccolò Longobardo (Niccolò Longobardi, 龙华民 [龍華民], lóng Huámín, 1565–1655) as members. Schreck died in 1630 and Longobardo eventually left, but they were replaced by the German Adam Schall (汤若望 [湯若望], Tāng Ruòwàng, 1592–1666) and the Italian Giacomo Rho (羅雅谷, Luó Yǎgǔ, 1592–1638). The Calendrical Bureau was eventually integrated into the Astronomical Bureau.

When the Jesuits started their work in China, they used the old Ptolemaic system, but after 1629, they switched to the Tychonic system. Schreck had been a member of the Cesi Academy (Accademia de' Lincei) with Galileo, and for eight years he wrote him repeatedly for help. The Pope had forbidden Galileo to promote his views, and even though Schreck promised that he would keep any help secret, Galileo was understandably not very eager to help the Jesuits. Galileo had a lot of arguments with two Jesuits astronomers in Europe, Horace Grassi and Christopher Scheiner, and eventually in 1624 he said that he had nothing to send to China.

In 1623 Schreck had instead decided to try to contact Johannes Kepler (1571–1630), and wrote to a friend asking to pass his request on to Kepler. To make him more interested in helping them, the Jesuits sent him data about old Chinese eclipse observations. Kepler was a Protestant and had studied theology intending to become a Lutheran minister. However, when the mathematics teacher at the Protestant Seminary in Graz passed away, the teachers at the University of Tübingen asked Kepler to give up his theological studies just months short of graduation and become a math teacher. This was a period of bitter religious conflicts in Europe, culminating with the Thirty Years' War (1618–1648). Kepler was forced to flee from Graz in 1600, from Prague in 1612 and from Linz in 1626 and barely escaped the Counter-Reformation in Sagan in 1628. However, when he finally received the letter from Schreck in 1627, he did not hesitate for a moment about helping the Jesuits, and already the next month he sent a lengthy letter to China, answering their questions in details and including several of his books and his brand new Rudolphine Tables.

When Schreck died in 1630, Adam Schall became their leader. In 1644 he went to the new Qīng rulers and presented his calculations for an upcoming solar eclipse on September 1. At this time the Manchus were suspicious of the Chinese, but Schall told them that they could trust him, because he was a foreigner like them. This worked, in spite of the fact that other Jesuits were busy supporting Míng loyalists in the South. He challenged the Chinese and the Muslim astronomers in the Bureau, and again the Jesuits' calculations were best. Schall was appointed director of the Astronomical Bureau, and the next year he formulated the current rules for the Chinese calendar in the Shíxiàn Calendar (时宪 [時憲]).

Jesuits were not supposed to take secular positions and strong factions within the Catholic Church, including the rival Dominican Order, were critical of what they saw as appeasement of Chinese superstition and heresy. At first Schall resisted his appointment, but eventually found a way to justify it to his superiors and accepted.

He became good friends with the Shùnzhì Emperor (顺治 [順治], 1638-1661, ruled from 1644), and became a mandarin of the first grade, first division. He was hoping to convert the Shùnzhì Emperor, but when the emperor realized that he would have to give up all his concubines and have only one wife, he lost interest in Christianity and started spending more time with Buddhist monks.

The fortunes turned for Jesuits, however, when the Shùnzhì Emperor died in 1661. A Chinese official, Yáng Guāngxiān (杨光先 [楊光先]) had as his slogan that it was "better to have a wrong calendar than to have foreigners in China". Yáng had several complaints against the Jesuit. The new calendar had two zhōngqì's in the 11th month of 1661, something that was impossible under the old system. Both the month after the 7th month and the 12th month had no zhōngqì's. The first was a 7th leap month, but the 12th was a fake leap month. Fake leap months did not exist under the old system, and it upset conservatives like Yáng. In the new calendar, the 11th month had three jiéqì's in 1661, something that was only possible in the new system. (In fact, the last jiéqì should have been in the following month, because it occurred 39 minutes after midnight, but the Jesuits made an error.) Schall had also presented the emperor with a calendar for the next 200 years, and Yáng claimed that this was improper since the emperor was blessed with infinite reign. Schall was also accused of not having followed the traditional ritual of "watching for the ethers" to determine the time of the beginning of spring ([23]). Finally, Schall was accused of having picked an inauspicious date for the funeral of a son of the Shùnzhì Emperor and of having cast a spell causing the early death of both the mother and the emperor. This accusation made a big impact on the Manchus, with their strong shamanistic culture.

Yáng won the support of Oboi (鰲拜, Áobài), one of the regents, and managed to have Schall, the Belgian Ferdinand Verbiest (南怀仁 [南懷仁], Nán Huáirén, 1623–1688), and two other Jesuits arrested in 1664. This was the start of the famous "Calendar Case". Unfortunately, Schall was struck with paralysis, and the burden of defending the Jesuits fell upon Verbiest. A solar eclipse was coming up on January 16, 1665, and while in prison, the Jesuits predicted it would occur at 3 p.m., Yáng predicted 2:15 p.m., and the Muslim Wú Míngxuǎn (吴明烜), also known as Wú Míngxuàn (吴明炫), predicted 2:30 p.m. On the day of the eclipse, the Jesuits were brought into the palace in chains, and everybody watched as the eclipse occurred at 3 p.m. sharp (14:59:54 according to computations by Salvo De Meis), exactly as the Jesuits had predicted! Unfortunately, the regents were not impressed and on April 15, the Jesuits were sentenced to death. However, the next day a strong earthquake struck Beijing and caused a fire in the Imperial Palace. Together with a comet that had appeared on April 13, this was taken as a sign from Heaven that the sentence was unjust. The Empress Dowager also called for the release of Jesuits, reminding them that Schall had been a close friend of the Shùnzhì Emperor. On April 19, the sentence of the Jesuits was converted to flogging and eventually to house arrest. The death sentence of five of their Chinese assistants, however, was upheld and carried out. In 1666, Schall died while still in house arrest.

In 1668, the Kāngxī Emperor (康熙, 1654–1722, ruled from 1661) was beginning to assert his authority and take over from the regents. Yáng and Wú complained that they could not find any competent astronomers, but an official who was supportive of the Jesuits told the emperor that the Jesuits were still in Beijing. The emperor sent a copy of the calendar made by Yáng and Wú to Verbiest on Christmas Day and asked for his comments. This was what the Jesuits had been waiting for four years, and their come-back is vividly described in Verbiest's book ([18]). When Verbiest pointed out several errors, including an incorrect leap month in the upcoming calendar for 1669, the emperor called a meeting with his officials, Yáng, Wú and the Jesuits. Verbiest described the errors in the Chinese calendar, and the emperor asked how if he could prove his claims. Verbiest offered to show that his computations could be verified by observations, and for the three next days, Verbiest successfully marked out the length of the shadows cast by various gnomons at noon. This took place at the end of December, and the rapid motion of the long winter shadows must have made this quite a dramatic event! Yáng was not an astronomer, and was against the Jesuits for philosophical and racial reasons, and by now he had been humiliated so thoroughly that he did not even bother to take part in the contest. Wú had no idea about how to compute the length of the shadows, and it was clear that the tide was turning for the Jesuits.

The final show down between Verbiest and Wú came in a series of observations tests in February 1669. Verbiest demonstrated that the Sun had reached the beginning of spring (立春, lìchūn), the beginning of spring, at noon on February 3 and marked the position of Jupiter and Mars that evening. He then marked the position of the Moon on the evening of February 17 and showed that the Sun had reached Yǔshuǐ (雨水) at noon on February 18.

They were to leave their sealed instruments pointing towards the predicted spot in the emperor's garden two weeks in advance. Verbiest easily beat Wú in the tests, and the emperor appointed Verbiest as director of the Astronomical Bureau, while Yáng and Wú were arrested. The emperor ordered by decree to move the leap month from after the 12th month of 1669 to after the second month of 1670, and to introduce the Western system of calibration, 360 degrees in the circle instead of $365\frac{1}{4}$, and time keeping, 96 quarters in a day instead of 100 kè (刻) ([18, p. 208]).

This was truly an incredible victory for the Jesuits. The promulgation of the calendar in the 10th month of the year was an important annual ceremony and about 2,340,000 copies of the calendar were printed each year ([45]). The death penalty was imposed on anybody who altered the calendar or printed an unofficial copy, and it was illegal to keep astronomical instruments or charts at home. Never before in Chinese history had a promulgated calendar been changed like this. It would be a giant loss of face to the Emperor, both domestically and internationally, since neighboring nations, notably Korea and Annam, also relied on the Chinese calendar. Verbiest was asked if he could find some way of covering up the problem, but he replied that in that case the calendar would be contradictory to the Heavens, and that made the court decide to move the leap month and amend the calendar.

Verbiest became personal tutor to the emperor, and even learned Manchu. Jesuits remained as directors of the Astronomical Bureau until 1746 and it was run by other Westerners until 1826.

There has been a lot of controversy over the contributions of the Jesuits. Let me sum up some of the main points.

Chinese astronomy was not necessarily inferior to Western astronomy. Guō Shǒujìng was a better astronomer than any of the Jesuits. However, the Chinese astronomers of the 17th century were conservative, and did not fully understand the methods they were using. In particular, they were not able to improve and correct their methods.

Because of the Catholic Church's condemnation of Copernicus and Galileo, the Jesuits did not dare to introduce the Copernican theory to China until 1761. Some critics argue that this was an attempt at keeping China backwards. This is a complex issue, but it is important to realize that when it comes to calendar making, the underlying theory is not that important. Accurate observations and computational skills are more important. In fact, one of the main advantages the Jesuit had was that they knew about logarithms, which enabled them to use more complicated numerical methods. They first used logarithms in 1633 ([10, p. 96]). It is also important to realize that after 1629, the Jesuits used the Tychonic system, and not the Ptolemaic system, as Needham claims.

The main change that the Jesuits implemented, was the change to using the true Sun. It is important to realize that this was a very important factor in the "Calendar Case". Since the fourth century BCE, Chinese astronomers knew that the motion of Moon was irregular, and since in the sixth century they knew that the Sun's motion was also irregular ([5]). They had started using the true Moon in the calendars since 619, but until the time of the Jesuits, they continued to use the mean Sun. I think there are several reasons for this difference. Using the mean Moon created discrepancies that were noticeable to everybody, but the errors caused by using the mean Sun were only noticeable to astronomers. Given the computational complexities associated with using the true Sun, the Chinese astronomers choose to stay with the old method. The Jesuits, however, needed to demonstrate their superiority. Changing the calendar by using the true Sun was a great way of making themselves essential, and making the Chinese astronomers look bad. It is therefore somewhat ironic that this backfired on them in that part of the reason why they were thrown in jail was because Yáng accused them of having made an error because their calendar did not agree with the old system! However, in the end it would help them again. When in 1668 the Kāngxī Emperor allowed the Jesuits to challenge Yáng and Wú, one of the objections Verbiest had was that Yáng and Wú had put a leap month after the 12th month. This was correct according to the old rules, but not according to the rules laid out by the Jesuits. When the emperor wanted to judge based on comparing the calendars with the motion of the true Sun, the Chinese astronomers were put at a serious disadvantage. Even if their methods were correct, they were aiming to follow the mean Sun, and not the true Sun. It was therefore not surprising that the Jesuits won easily. In fact, part of the reason why the Jesuits won, was because they had moved the goalposts!

It is of course true that the Jesuits had ulterior motives; their goal was to win converts. But in spite of this, I believe that the Jesuits made a positive contribution to China, and contributed immensely to understanding between China and the West. And from a mathematical point of view, they made the Chinese calendar much more exciting!

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